The Bible in War

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THE RIFLE IN WAR

—BY—

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PREFACE TO SECOND EDITION.

A few years ago the artillery was generally regarded as an abstruse and occult part of the military profession in which the officer had to possess a knowledge of such difficult mathematical calculations and such deep and involved theory that only a very few, and those the very brightest minds of all could hope to fathom its mysteries. It was so deep and difficult that many of its own officers did not understand it and put the questioning alien off with a wise wave of the hand and an intimation that it could not even be explained. And yet at that very time, the problem of the use of the fire of small arms was much more complicated and as little understood. The artillery experienced a renaissance about twenty years ago and developed from a technical into a tactical arm and "Modern Artillery" was an accomplished fact. Having but four or six guns to handle, they have worked with such skill and understanding upon the problem of the use of those guns that the whole art of war has been all but tumbled over by the efficiency of their fire. One of the foremost of the workers in this fertile field was Lieutenant General H. Rohne and much of the progress in the artillery world is due to his lucid and remarkable works on that subject. Some ten years ago, this gifted officer turned his attention to the more complicated and more difficult task of the fire of infantry, and working along lines made familiar through his artillery investigations he devoloped the study of the rifle as we know it to-day.

Prior to von Rohne, Parravicino, a brilliant Italian General had begun a series of experiments and had laid a broad foundation upon which General Rohne later was to build the finished structure. After the appearance of von Rohne's first books, the French investigators entered the field and added much to the knowledge of the world on the subject, but France had become so wedded to the theory of Woloskoi, a Russian General of renown and an author of several books on the subject of the rifle in war, that although the French investigators produced their full share of the best modern books on the subject, they were not able to influence their own army to the same extent as General Rohne influenced that of Germany. As a result of General Rohne's works, the German Firing Regulations are unquestionably the best example of what a system of firing instruc-

tion should be (for German conditions, of course) and there are many points in that system which we must in time take up and incorporate in our own, not "because they are German" but because they are built upon a solid foundation of proven theory rather than the result of chance or of the casual selection of the "board."

Some two years ago, the author undertook the preparation of a suitable text for use in the Service Schools at Fort Leavenworth, and published a portion of "The Rifle in War." This first edition was prepared under a considerable stress of other work and while very incomplete and fragmentary served its purpose as a basis for his conferences on this subject with the students of the several schools. The need for a more permanent and complete presentation of the subject, however, led to a revision and completion of the original edition and although the author's work as an instructor in two departments interfered with the proper preparation of the text of the revision to a very considerable extent, it was at last accomplished.

To Major John F. Morrison, General Staff, Assistant Commandant Service Schools and Senior Instructor, Department of Military Art, the author acknowledges a debt of gratitude for the encouragement which he received from that officer and for his valuable criticisms on the manuscript. The lack of a suitable text-book in the English language on this very important subject forced the author to prepare one for his own use, and the continued lack of such a book is his justification for submitting this text to the army for its use until a better one shall have been published.

In the following pages, much will be found that appears didactic and much that appears pure theory with no practical end in view; but to attain a thorough knowledge of this, as of any other subject, it is believed necessary that the student should start at the bottom, lay a firm and broad foundation of elementary knowledge, and, upon this, rear the completed structure which shall have a war value. In doing this it is inevitable that some theory should be included and that some didactic statements should be made.

The use of mathematics is as limited as is consistent with an understanding of the subject and only elementary mathematics are used. Further, the author, mindful of the fact that the subject is a study of rifle firing rather than of mathematics, has made many long leaps from the statement of the problem to the calculated result, seeking to impart a lesson and to point a moral rather than to carry the student through the sometimes long computations by which the result was obtained. The student is, therefore, able to read and grasp the lesson sought to be imparted without losing the thread of the argument by a digression into other fields.

There are certain elementary principles with which the student is presumed to be familiar and, as a rule, only explanations and definitions are given such as are not commonly found in text books on elementary musketry. The method employed of comparing the size of the target and the relative dispersion is that which has for years been the basis of inquiries into the effect of fire by artillerists and is now used by all scientific infantrymen for reasons that will be made apparent.

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BIBLIOGRAPHY.

In the preparation of this text, many books were consulted, a complete list of which is not believed to be necessary. The most valuable of these books however, are given in the following list, most of which are as yet untranslated.

Von Rohne:	"Schiesslehre für Infanterie" u. s. w.
	"Das gefechtsmässige Abteilungsschiessen der Infanterie, und das Schiessen mit Machinengewehren" (4th Edition).
	"Neue Studien über die Wirkung des Infanterie-Gewehrs beim gefechtsmässigen Abteilungsschiessen."
	"Die Taktik der Feldartillerie f. d. Officiere aller Waffen."
Anon:	"Das Kriegsmassige Infanterieschiessen" (Mittler-Berlin 1903).
Lichtenstern:	"Schiestaktik der Infanterie."
Lamiraux:	"Principles of Fire."
Langlois:	"L'Artillerie de Campagne en laisson avec les otre Armes."

While there is no book in the English language comparable to those above quoted in the fundamental principles of fire effect, the following translations and original English works are pertinent to the subject as dealing with the use of infantry fire in battle:

Honig:	"Inquiries into the tactics of the future."
Balck:	"Modern European Tactics" (Vol. I). This is the best book in English on the subject of fire tactics. It is the only volume of Balck's whole masterpiece which has been translated.
Maude:	"Notes on the Evolution of Infantry Tactics."

In addition to the foregoing, reference is made in the text to other authors and books which have been consulted, and the student who is inclined to follow up the subject by collateral reading will find that one book will suggest another, some of which will be of value and others of no particular use for his purpose. To prevent

the loss of time in reading the latter class of books, the foregoing list of the "best" books is given. The English classics written by Colonel Mayne are too well known to need recommendation. His latest book, "The Infantry Weapon," however, is not developed along modern lines, but is a development of the older ideas as to the use of fire in battle and its study in peace. It is to be hoped that we shall, in time have a new book in English on this important subject from the pen of that gifted author.

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CHAPTER V.

Problems in Fire Direction.—Map problems. Tactical rides.

CHAPTER VI.

Instructional Methods.

ERRATA

Page 26.—Line 26. For "10" read "1.9".

Page 50.—Position of shot 3, on Fig. 18. should be near shot 7 (i. e. 11.20 and 6.55).

Page 51.—Square of M. V. Dev. should be 1.2769.
Square of M. H. Dev. should be 1.0609.
Sum of Squares should be - - 2.3378.

Page 71.—Line 2. For "center" read "bottom."

Page 72.—Line 22. For "impacct" read "impact."

Page 74.—Third line from bottom. For "20.7" read "20.2".

Page 74.—Second line from bottom. For "20.7" read "20.2".

Page 75.—First line. Should read "=44 inches. 44÷20.2=2 18=85.9; 85.9÷2=43.0".

Page 75.—Second line. Should read "percent. 50—43=7 per cent".

Page 75.—Line 7. For "examples 3 and 4" read "examples 8 and 9".

Page 76.—First line. Should read "40÷27.4=1.46=67.3; 44÷20.2 =2.18=85.9; 85.9".

Page 78.—Line 12. For "14.6" read "8.75".

Page 87.—Line 29. For "how many" read "what".

Page 91.—Lines 14 and 17. For "11.949" read "7.342".

Page 93.—In foot note. In first line, for "40" read "35.4"; in fifth line for "40×0.36=1.44 per cent." read "35.4× 0.36=12.64 per cent."; in seventh line, for "40×0.03 =1.20 per cent." read "35.4×0.03=1.06 per cent.".

Page 101.—Line 17. For "$S=56\times\frac{1}{2\text{-}1}$" read "$S=6\times\frac{2}{2\text{-}1}$".

Page 105.—Fig 41. For "c×cot. f°" read "c×tan f°".

Page 111.—In line 24, for "(17.9)" read "(19.7)"; in line 25, for "50" read "49"; in foot note, line 1, for "valve" read "value" and in line 2, for the first "d" under the radical sign read "$\frac{d}{v}$".

Page 112.—In line 1, for "17.9" read "18"; in line 2. for "50" read "49".

Page 117.—Line 2. For "10.5" read "21.4".

Page 128.—Line 21. For "2.2" read "8.2".

Page 129.—Lines 4 and 34. For "2.2" read "8.2".

Page 130.—Line 10. For "average" read "good".

Page 132.—In third column of Table VI—400 yard line—for "88.8" read "68.8", and—500 yard line—for "58.8" read "48.3".

Page 136.—Line 14, for "third" read "half" and in line 15, for "(15:51.3)" read "(15:34.4)".

Page 137.—In line 7, for "34" read "24" and in line 12, for "less" read "greater".

Page 140.—Line 16. For "six" read "three" and for "(14.5)" read "(7.5)".

Page 146.—In line 21, for "308" read "305"; in line 22, for "135" read "125", and in line 24, for "308" read "305."

Page 151.—In column 1 of the Table, transpose "1050" and "950"; in column 3—in line opposite "Combine" for "0.7" read "1.7" and opposite $\left.\begin{matrix}950\\1000\\1050\end{matrix}\right\}$ for "0.5" read "1.1".

Page 152.—In line 16, for "35" read "2" and in line 17, for "two-thirds" read "four-fifths".

Page 154.—Fifth line from the bottom. for "3 yards" read "3 feet".

Page 158.—Lines 12 and 13, for "the figures at two yards interval" read "one figure on each two yards of front".

Page 159.—Line 11. Strike out the words "but" and "of".

Page 160.—In line 14, for "22" read "20"; and in the last column of the table,—Line of Sections—for "220" read "200", for "158" read "143" and for "122" read "111".

Page 161.—In the last column of the table—Line of Sec. Cols.,—for "69" read "61", for "68" read "59" and for "67" read "59".

Page 188.—Line 7. For "1:13" read "1:7".

Page 190.—In line 18, for "63" read "64" in both cases; in line 24, omit the word "whereas"; in line 25. for "less" read "more"; in line 26, insert the word "which" between the words "disabled" and "with" and in line 27, for "with" read "from".

Page 204.—Line 22. For "is" read "are".

THE RIFLE IN WAR.

CHAPTER I.

THE TRAJECTORY.

When a rifle is fired, the bullet during its passage through the barrel follows the straight line of the axis of the bore; so soon, however, as it leaves the muzzle and is unsupported, gravity begins to pull it toward the earth, and it is then subjected to two forces acting in different directions, one forward, the other downward, which cause it to travel forward and downward along a curved path called the TRAJECTORY.

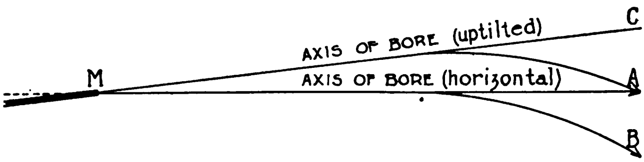


FIG. 1.

Thus, in Figure 1, if the barrel is horizontal, the forward impulse given the bullet by the powder explosion would make it travel along the horizontal line MA and strike the target at A. The downward pull of gravity, however, causes it to depart from this line and the bullet strikes at B, somewhere below the horizontal.

In order that the point A may be struck it is neces-

sary, then, to uptilt the barrel so that instead of being horizontal it shall point upward at C, a spot on the target as much higher than A as the bullet will fall in the time consumed in flight. (CA=AB). With the barrel pointed at C, the bullet will then strike at A instead of at B. The marksman wishing to "aim" at the point to be hit, rather than at some higher point, as C, the rifle is provided with a rear sight of such dimensions that the object to be hit (A) is perceived over the top of the front sight and through the notch of the rear sight when the axis of the bore is uptilted so as to point at C. The bullet will then be sent forward toward C by the force of the explosion and be pulled down by gravity so as to strike at the point of aim, or A.

The angle which the axis of the bore makes with the line of sight over the sights as described is called the ANGLE OF ELEVATION, and it is almost exactly equal to the angle which the tangent of the trajectory at the

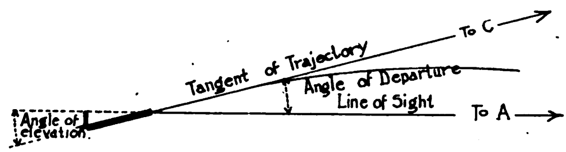


FIG. 2.

muzzle makes with the line of sight, which is called the ANGLE OF DEPARTURE.

The size of these angles is dependent upon the shape of the trajectory; it varies with the RANGE or distance from the muzzle to the target, and for any given range, it varies with the determining factors of the trajectory.

The form of the trajectory is influenced by:

1. The initial velocity,
2. The angle of departure,
3. Gravity,
4. The resistance of the air,
5. The rotation of the projectile about its longer axis.

VELOCITY.—The velocity of a moving body is its rate of motion. It is expressed by stating the distance passed over during some given time, or that would be passed over during that time if the uniform motion continued so long When velocity increases, it is said to be accelerated, when it diminishes it is said to be retarded.

Starting from a state of rest in the bore of the rifle, the velocity from the moment of discharge is accelerated up to a point about 35 feet in front of the muzzle, and beyond that point it is retarded. At a point 78 feet in front of the muzzle, the powder gases cease to act upon the projectile, and it is, therefore, at this point that the velocity is determined for any given lot of ammunition, the velocity at this point being called the INITIAL VELOCITY. The initial velocity of the Springfield rifle, '06 bullet, is approximately 2,700 feet per second, that is to say, this bullet would travel 2,700 feet in the first second, if the rate of motion remained unchanged that long.

Velocity is caused by the pressure on the base of the projectile, of gas generated by the combustion of the powder. This pressure furnishes a certain motive power, the force of which depends upon the amount of pressure and the distance which the projectile travels

in the bore under this pressure. The amount of the pressure is dependent upon the weight of the powder charge; the heavier the charge of a given powder, the greater the pressure, but the VELOCITY which a given pressure will develop depends upon the weight of the projectile.

There is, therefore, a relation or ratio between the weight of the powder charge and the weight of the projectile (called the RATIO OF CHARGE) upon which the initial velocity depends. In a general way it may be said that with a given projectile it is necessary to quadruple the charge to double the velocity.

The rapidity of combustion of the powder also greatly affects the amount of the motive power generated by a given weight of powder; and so the initial velocity. Powder will absorb a certain amount of moisture from the air and the more moisture thus absorbed the slower will be the combustion. Low temperature or an abnormally large combustion chamber will have the same effect, while the reverse conditions will increase the rate of combustion and the initial velocity. Place a chilled cartridge in a rifle and fire it. The powder will burn slower, the initial velocity will, therefore, be lower and the shot will strike low. Place a heated cartridge in the rifle and the rate of combustion will be increased, the initial velocity will be higher and the strike will be high. This is one of the reasons why we overshoot in summer and undershoot in winter.

Remembering that both gravity and the forward impulse are acting on the bullet as it speeds forward and that a bullet will reach the target sooner with a high initial velocity than with a low one, it is evident that the high velocity bullet will be subjected to the

action of gravity for a less time than a slower moving bullet, and that it will be pulled down by that force a less amount; consequently, that from a horizontal barrel, it will strike higher.

As two cartridges are never mathematically identical in the amount of moisture which the powder has absorbed, the weight of the charge and the temperature, it is seen that herein lies one of the reasons why two shots fired under seemingly identical conditions do not hit the same spot.

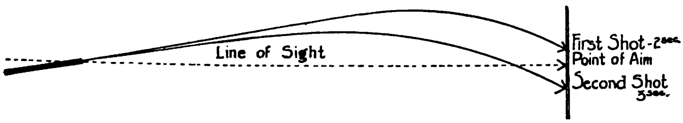


FIG. 3.

ANGLE OF DEPARTURE.—Of much greater importance than changes in the ratio of charge are the unavoidable differences made by the firer in the angle of elevation.

On account of the imperfections of the human eye, the lines of sight of successive shots are not identical; that is, are not directed at the same spot so that the point of aim varies with each shot (error in aiming). As a result of the shock of the discharge, the position of the axis of the bore changes, with reference to the line of sight at the moment that the bullet leaves the muzzle ("jump"), and the amount of this change varies

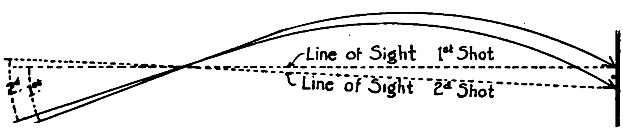


FIG. 4.

with each shot. In addition to the jump and the errors in aiming are the errors in holding; that is, the errors which are caused by the firer's inability to keep the line of sight correctly directed at the moment of discharge, and to keep the plane of the trajectory perfectly vertical. Each of these disturbing factors cause differences in the angle of departure of successive shots, and this is one of the most important reasons why successive shots under seemingly similar conditions do not strike at the same point.

GRAVITY.—Gravity imparts a velocity to a falling body which is constantly accelerated. In the first second, for example, the bullet will drop 16.1 feet, at the end of the first second it has a velocity of 32.2 feet per second, and during the second second it will drop 48.3 feet, attaining a velocity at the end of that second of 64.4 feet, etc. The amount of this motion in its several phases is shown in Table 1.

TABLE 1.—GRAVITY (Vacuum).

At the end of the — second.	*Velocity in ft. per second.*	*Distance Fallen since end of preceding second — ft.*	*Total distance fallen — ft.*
1	32.2	16.1	16.1
2	64.4	48.3	64.4
3	96.6	80.5	144.9
4	128.8	112.7	257.6
5	161.0	144.9	402.5

This increase in distance fallen causes the bullet to depart more and more strongly from the horizontal with every additional second or other unit of time consumed in its flight. The path of a bullet fired from a horizontal barrel is, therefore, a downward curve, compara-

tively flat at its beginning when the bullet is dropping at a slow velocity and becoming more curved as it is subjected to the influence of gravity for a longer time, since the velocity of gravity is continually accelerated.

Referring to Figure 5, if the bullet is travelling at a uniform velocity so that at the end of the first second it is at A, at the end of the second second it is at B, and at the end of the third second it is at C, it will have fallen 16.1 feet in the first second and be at A′, which is 16.1 feet below A; at the end of the second second it will have fallen 64.4 feet and will be at B′ (64.4 feet below B), and upon arriving at the target at the end of the third second it will strike 144.9 feet below the point C, or at C′. These distances ignore the effect of the atmospheric pressure and resistance, and

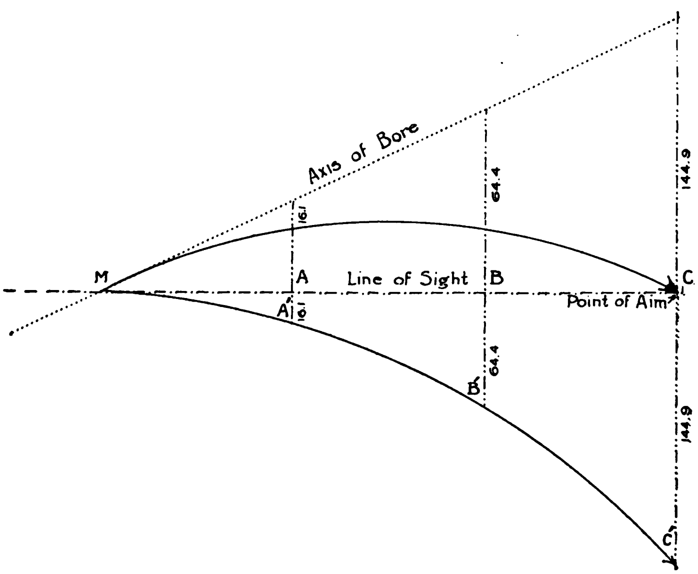


FIG. 5.

are altered, of course, so soon as these factors are included in the discussion.

RESISTANCE OF THE AIR.—A projectile passing through the air meets with a certain resistance which retards its velocity. The amount of this resistance is immaterial in this discussion, but it is worthy of note that the greater the velocity the greater will be the resistance offered by the air to its passage.

By a second law, the resistance of the air varies directly as the cross section of the projectile and inversely as its weight, that is, a projectile with a cross section of 2 square inches will meet with a resistance twice as great as one whose cross section is only one inch; but a projectile weighing 1 ounce will be retarded by the air resistance twice as much as a projectile having the same cross section area, but weighing 2 ounces. Every increase of weight, therefore, which can be gained without increasing the cross section of the bullet, is an advantage in that it suffers less retardation by the air in its flight.

The proportion which the weight of the projectile bears to the cross section is called the SECTIONAL DENSITY of the particular projectile, and in order that the full benefit of the high initial velocity of the rifle may be enjoyed, a material is chosen for the core of the modern bullet which is about 10 times as heavy as iron, thus assuring a great sectional density and a correspondingly low air resistance.

An increase in the absolute weight of the projectile is out of the question because of the recoil, which would be too severe for a hand-weapon.

The angle at which the air strikes the surface of the projectile is also of great importance, therefore the

shape of the bullet is made a matter of great study and experimentation. Generally speaking, the sharper the point the less the resistance, and it is known that a tapering of the rear part of the projectile reduces the resistance as it facilitates the escape of the gases compressed in front of the bullet.

There are, however, objections to the tapering of the base which make it inadvisable to take advantage of this resistance-lessening device, although the German "S" bullet is so shaped. Any roughness on the exterior surface of the projectile increases the friction of the air, hence a bullet without grooves will suffer less resistance than one with grooves or channellures, and a steel jacketed bullet with its smooth surface offers less frictional resistance than a lead surfaced bullet.

The density and movement of the air also affect the amount of resistance of the air. The denser the air, the greater the resistance. A high temperature, a low barometer and dry air all tend to decrease the density of the air and so to reduce the air resistance. As before stated, in Summer we generally overshoot and in Winter undershoot, and since hot, dry air is more often met in Summer than in Winter, this is another reason why this is so. The effect of the wind increases as its velocity, and depends upon its direction. A side wind causes the bullet to move sideways; blowing into the face of the firer, it increases the resistance of the air, and blowing in the direction of the fire, it reduces the resistance.

ROTATION OF THE BULLET.—The twist of the rifling imparts a rotary motion to the projectile around its longer axis, which, on account of this rotation, retains motion also affects the motion of the bullet in that it

its position generally parallel to the tangent of the trajectory throughout its flight instead of remaining in the angle at which it left the bore or instead of tumbling over and over, as it would without this axial rotation.

Always travelling point on, the bullet presents the minimum of surface to the air in its passage, and the influence on the trajectory of this cause of deformation is consequently kept down to the minimum. The rotary causes the whole path of the bullet to be deflected sideways in the direction of the twist (to the right, therefore, in the Springfield rifle), but the amount of this deflection—it is called the DRIFT—is negligible in collective firing both because it is relatively small and because it is in a large measure corrected automatically by the design of the rear sight leaf.

THE TRAJECTORY.—The path through the air which the bullet follows in its flight is called the TRAJECTORY, and under the combined effect of the influences just discussed, the trajectory when firing with a horizontal line of sight, first rises above the horizontal with a scarcely perceptible curve to a point a little more than half way to the target called the SUMMIT, beyond which point it curves downward with ever increasing curvature until it strikes the target or the ground. The point where the line of sight meets the target is called the POINT OF AIM, and that where the bullet (trajectory) meets the target is the POINT OF IMPACT. Theoretically, the point of aim and the point of impact should coincide; practically, because of some one or more of the influences dismarksman and the greater the perfection of the arm and cussed, it seldom does. The greater the skill of the its ammunition, the more often will the two points coincide.

In computing or determining the dimensions of the trajectory, the line of sight is assumed to be horizontal, and at regular intervals (usually at 100 yards) the height of the trajectory above the line of sight is measured and recorded as an ORDINATE of the given trajectory at the given distance from the muzzle, this distance being called the ABSCISSA. The highest ordinate is that of the summit whose abscissa is a distance a little greater than half of the range, as stated.

That part of the trajectory between the muzzle and the summit is called the RISING BRANCH OF THE TRAJECTORY, while that beyond the summit is called the FALLING BRANCH OF THE TRAJECTORY. It is with this latter that we are most interested, as it contains the target and the ground in its vicinity.

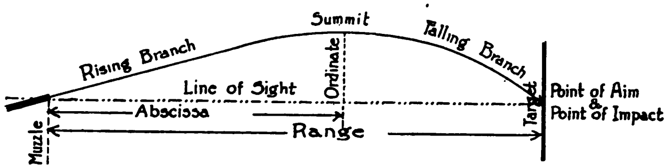


FIG. 6.

If a man (standing) fire a rifle with a horizontal line of sight, it is evident that for a certain distance in front of the muzzle another man may be hit by the bullet while in the rising branch of the trajectory. If we assume that the muzzle is at a height of 56 inches from the ground, and that the average height of a standing man is 68 inches, then it will be dangerous for a man to stand anywhere in the line of fire between the muzzle and the point where the bullet, in its upward flight, rises above

the head of the man (68 inches). Since the line of sight is 56 inches above the ground, the trajectory will be 68 inches above the ground when it has risen 12 inches above the line of sight, or when it has attained an ordinate of 12 inches. If the firer is using a great angle of elevation, the trajectory will attain an ordinate of 12 inches in a rather short distance; if, on the other hand, the angle of sight be small, the bullet will travel so nearly horizontal that it will not attain the 12 inches in a very great distance, if at all.

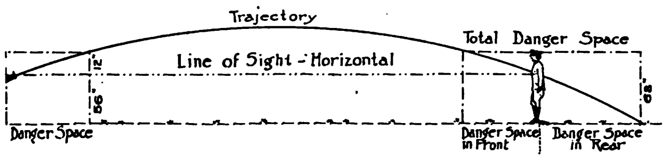


FIG. 7.

Whatever this distance is, in front of the muzzle, within which the bullet does not rise higher than the object, it is called the danger space of the rising branch of the trajectory. If the firer is shooting from a prone position, so that the muzzle instead of being 56 inches from the ground, is only 12 inches, then the space would be dangerous for a standing man from the muzzle to a point where the ordinate of the trajectory is (68″—12″=) 56″ high and the danger space in this case will be about five times as great as in the case of a standing firer. So also is the extent of the space dependent upon the angle which the line of sight makes with the horizontal, as where the firing is done up or down hill, where a prone man aims at the head of a mounted man, etc. The danger space for a mounted man is, obviously, great-

er than for a man lying down, and so in each case, it is necessary to know all of the conditions before a table of danger spaces, as published by the Ordnance Department, can be of value.

In the falling branch of the trajectory is another danger space analogous to that just discussed, and, like that, dependent upon certain conditions of firer, line of sight and object for which the space is dangerous. If the point of aim is taken at the head of a man standing, all of the danger space will lie in the rear of the target; if, on the other hand, it is taken at the foot of the target, the entire danger space will lie in front of the target, if, as in the case usually assumed, the point of aim is at the middle of the object fired at, part of the danger space will lie in front of the target and part in rear.

If a rifle is fired a great number of times under conditions as nearly uniform as it is possible to make them, the rifle being fired from a fixed rest so as to exclude the human errors of incorrect sighting, holding, etc., the bullets striking the target will group themselves about a central point called the CENTER OF IMPACT, and form a circular or elliptical group whose dimensions and shape for any gun vary with the distance of the target from the firer.

At the muzzle all of the bullets will, of course, pass through the same point. If a paper screen is placed 100 yards in front of the muzzle, it will be perforated by all of the bullets within a circle four or five inches in diameter. At 200 yards, the circle enclosing the shots will be about 10″ or 11″ in diameter, and so on, the circle constantly increasing in size with the range. A line connecting the centers of impact of all these

circles is called the MEAN TRAJECTORY, and a cone containing all of the circumferences of the circles would mark the limits of the SHEAF or CONE OF TRAJECTORIES.

The more nearly perfect the gun and ammunition, the smaller will be the size of this sheaf. As the mean trajectory is the average trajectory, it is computed or determined for the service rifle and is simply called the *"trajectory,"* all ordinates are computed to this mean trajectory and the angles of departure and fall refer only to it.

The pattern on the target made by all of the bullets is called the SHOT GROUP. If received on a vertical target, it is a VERTICAL SHOT GROUP; if on a horizontal target, it is a HORIZONTAL SHOT GROUP.

Since the angle of fall is known, the size and shape of a vertical shot group or any part of it can easily be converted into a horizontal shot group and *vice versa* through the use of natural tangents.

When, instead of one rifle fired 100 times and from a rest, we fire one shot each from 100 rifles held by men as in actual service shooting, the size of the sheaf becomes very much larger. The better trained the marksmen are, that is, the "closer" they shoot, the smaller the sheaf will they produce and the smaller the resulting group. With poorly trained men, the group sometimes assumes very large proportions indeed, and becomes quite useless as an offensive factor in war.

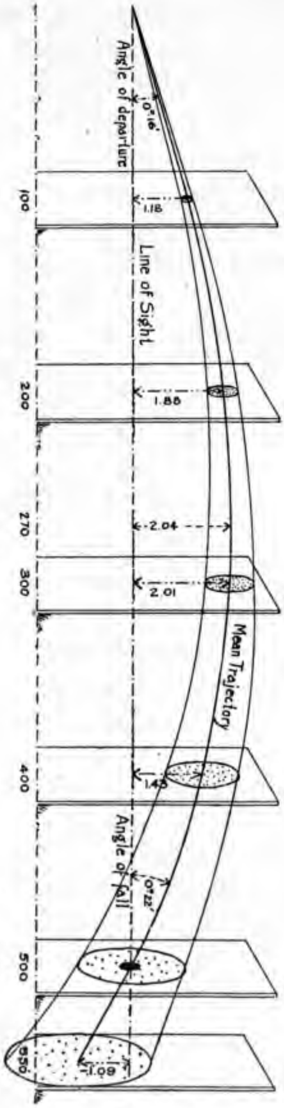

Angle of departure
0°16'
Line of Sight
Mean Trajectory
Angle of fall
0°22'
100
200
270
300
400
500
550
1.18
1.88
2.04
2.01
1.43
1.09

FIG. 8.

Of all the deforming factors those discussed under the title "Angle of Departure" are the most potent, and it is only by diligent training and practice that these "human" errors can be reduced. The soldier is, therefore, taught to shoot, aim and hold accurately so as to form as small a sheaf as possible, and the officer is taught how to direct this small collective sheaf so as to assure the highest returns in killed and wounded for the excellence in marksmanship of the soldier.

However instructive as an elementary study an inquiry into the niceties of shooting—the accuracy of a single gun, etc.—may be, it is the collective fire of masses of men that must occupy the attention of the officer in war. It is for this reason that so much study is spent upon an investigation of the characteristics of the collective sheaf which in the remainder of this text will be termed simply, "THE SHEAF," "collective sheaf" being understood.

When a group of marksmen fire either simultaneously or "at will" with the same point of aim and the same elevation, the bullets will form a shot group having the general shape of an ellipse, with its major axis vertical, and will be symmetrically grouped about the center of impact—not necessarily about the point of aim. They will be grouped more densely near the center of impact than at the edges and ½ of all the shots will be found in a strip about one-fourth of the size of the whole group.

If the whole group is found to be say 40 inches high, then a strip about 10 inches wide, placed so that about 5 inches lie on each side of the center of impact, will contain 50% of all hits. The width of this strip is called "THE MEAN, or 50% DISPERSION"; vertical if

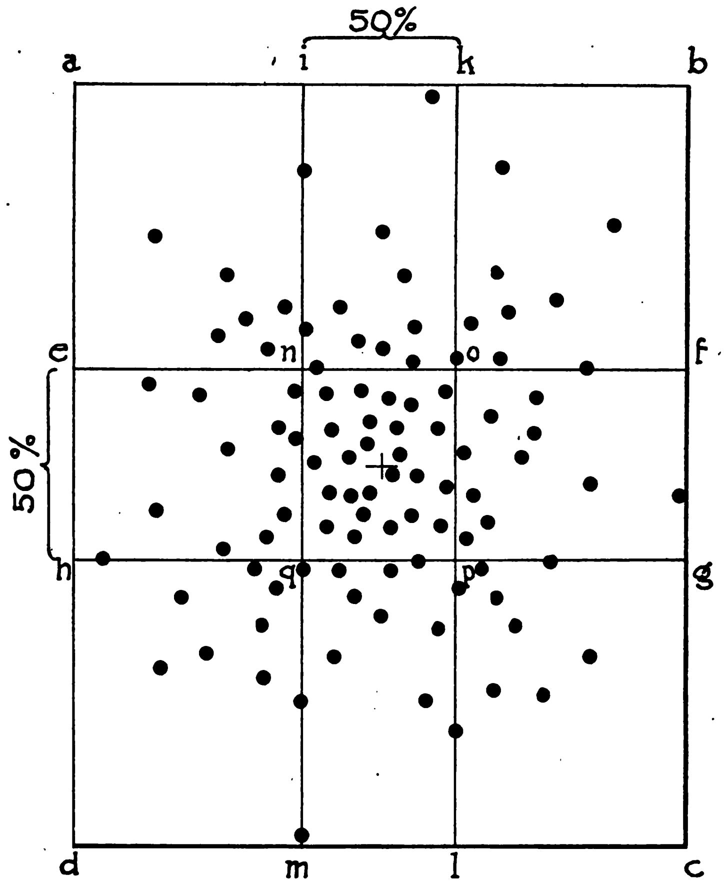


FIG. 9.

measured vertically, lateral if measured laterally. When considering the horizontal shot group, the mean lateral dispersion retains its same significance; but what on the vertical target is called the mean *vertical* dispersion is

known here as the mean longitudinal dispersion and is measured in the direction of the line of fire. As in the whole group, the mean vertical dispersion and the mean longitudinal dispersion are mutually convertible by using the tangent of the angle of fall.

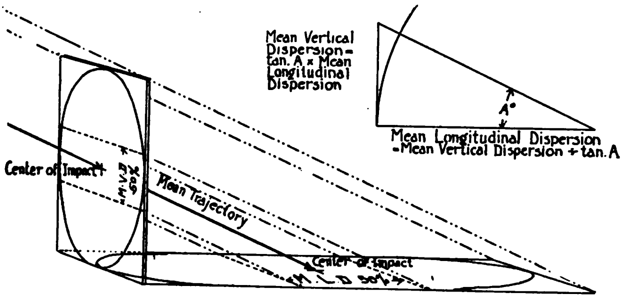


FIG. 10.

With a shot group of fixed dimensions, it is evident that when the target is made sufficiently large, every bullet fired will hit the target.

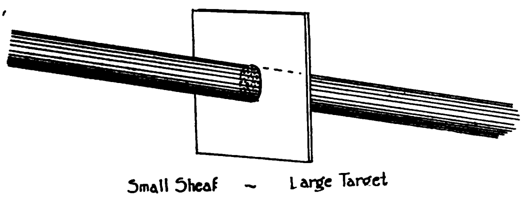
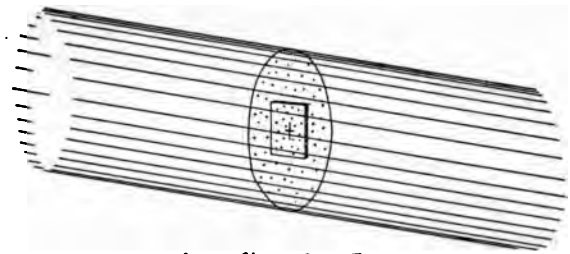


FIG. 11.

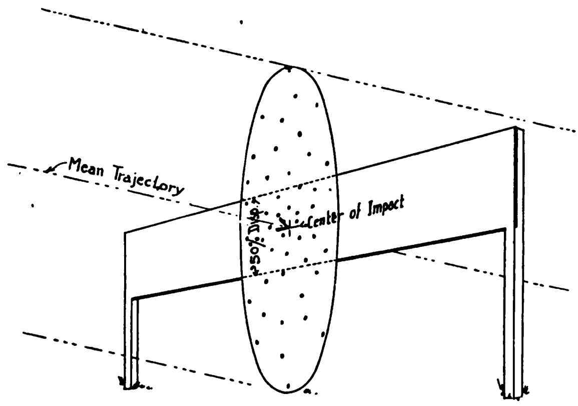
It is also evident that with a very small target, only a portion of the bullets will hit the target, the others missing high, low, and to either side.



Large Sheaf ~ Small Target

FIG. 12.

When the target is of great width and is as high as the mean vertical dispersion, ½ of all bullets will hit it, the others missing high or low.



Wall Target, very long and exactly as high as the Mean Vertical Dispersion ~ Center of Impact in center of Target

FIG. 13.

There is, thus, a relation between the size of the target and the size of the shot group, which has been

tabulated for convenience in solving questions of probable effect of fire, the values of which are called "PROBABILITY FACTORS," and by means of this table we may examine and compare the results to be anticipated in the many cases arising in any discussion of fire effect. This table is given on page 47. The subject will be found more fully explained in Chapter II, but it is sufficient for present purposes to know that probable effects *can* be foretold and that by foretelling the results in two contrasting cases the value of any given influence on shooting can be ascertained.

For example, 90 hits out of 100 shots may be good shooting against a certain target at 200 yards, and 9 hits out of 100 shots be better shooting at 1,000 yards against the same target. Or, again, a group of marksmen shooting at 800 yards range, but using, by order of their commander, an elevation of 1,000 yards, may make 5 hits; when, without any increase of accuracy on the part of the firers, 8 or 10 hits might have resulted had a correct elevation been used.

In order that comparisons in shooting may be made, in order that the influence of an incorrect elevation may be demonstrated, and for other and analogous reasons, the whole subject of war shooting is based upon an understanding of dispersions, misplaced centers of impact, densities and such fundamental principles. The merest tyro knows that it is harder to hit a small object than a large one, harder to hit an object at long range than at short, harder to make a high percentage on the range with the distance estimated than when the distance to the target is known; but this is insufficient, one must know how to foretell a result under perfect conditions,

then, from this, to measure the degree of importance to be attached to every deviation from these perfect conditions. The statement that expert riflemen will not make as many hits in war as will average shots, at first glance seems unreasonable; yet it is undoubtedly true and may be proven by actual tests anywhere and by anyone, as it has abundantly been proven in the past. We know that good shooting will result in more hits, as a general proposition, than poor shooting. How, then, is this knowledge to be reconciled with the other knowledge that "the better you shoot the less you hit"? And why, if this is so, should instruction in firing be given at all? The explanation is simple. The more highly trained the marksman and the more perfect the rifle, the smaller will be the sheaf produced. If this sheaf is properly directed so that the center of impact is in the center of the target, a very much higher percentage of hits will result than can be made by poorer marksmen with their larger sheaf

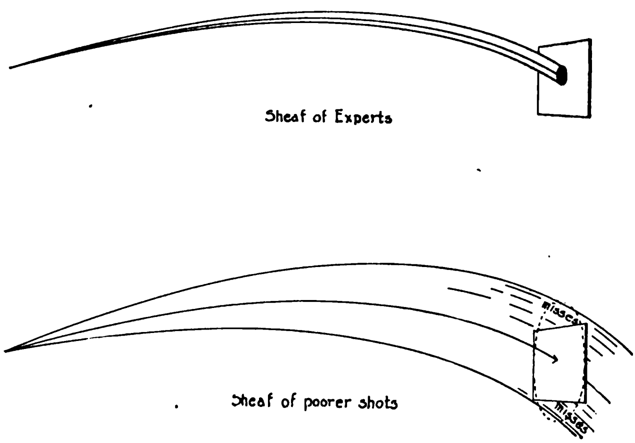


FIG. 14.

or dispersion, for the squad of experts will put all of their bullets in the target while the squad of poorer shots is making some misses.

If the center of impact is not properly placed, for example if it is a foot or so above the target, the close shooting expert squad will miss the target with every bullet, while the poorer shooting squad is still getting some hits upon the target.

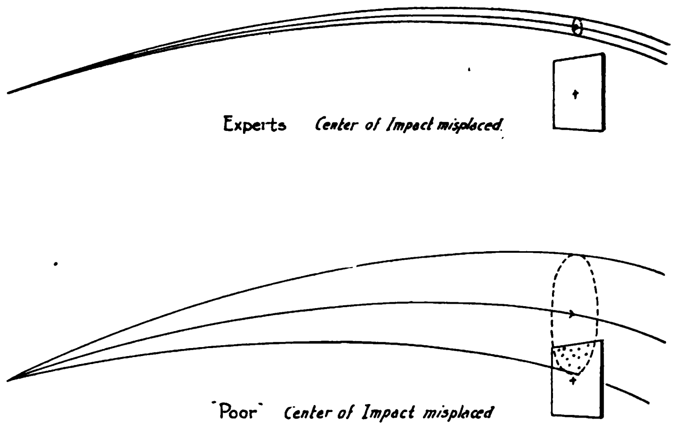


FIG 15.

When it is remembered that the placing of the center of impact is not a question of aiming but the result of an ordered angle of sight (range), and that in war, ranges are estimated—and usually very poorly—it will be seen that in the usual case, more hits will result from average shooting than from very fine shooting. That marksmanship has nothing to do with the location of the center of impact is easily demonstrated by reference to a table of ordinates. Examining an extreme case, let us suppose that the range is actually 500 yards, but is estimated at 1,000 yards, the firing being over a level plain.

The line of sight is horizontal and is not changed.

by an incorrect estimate of the range. *With a correct elevation,* the trajectory rises to its summit and then falls, meeting the line of sight at the target where point of aim and center of impact should coincide. *With sights*

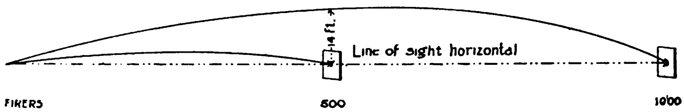


FIG. 16.

set at 1,000 *yards,* the trajectory is still rising at a range of 500 yards and is 14 feet above the line of sight and point of aim.

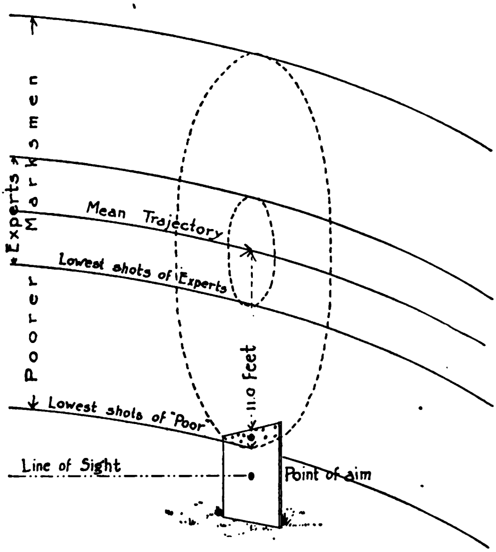


FIG. 16a.

If the target is 6 feet high, and the point of aim is at the center of the target, the mean trajectory will

be 14 feet above the center of the target, and 11 feet above the top edge of the target. With a cone or sheaf 22 feet in diameter, so that the lowest bullets are 11 feet below the center of the sheaf (mean trajectory) it is evident that these lowest bullets will just graze the top of the target, making no hits at all. And this displacement of the center of impact would be the case without any regard at all to the skill of the firers, the only result of skill in shooting manifesting itself in the size of the cone. With a close-shooting expert squad of marksmen the cone or sheaf would be very small (about 6 feet in diameter), and no hits at all could be expected. With poorer shots (assumed dispersion such as to produce a cone 23 feet in diameter) a few scattering shots would be expected. The point of aim in each case would be the same *and would not influence the result;* but the shooting skill of the marksmen would affect the result the reverse of what would be expected by anyone unfamiliar with the subject.

It will be observed that two factors here enter into the probability of hitting: (1) the skill of the men; (2) the skill of the officer in estimating distance.

If an officer cannot announce the elevation with reasonable accuracy to his men, it is self-evident that his organization will attain results in war in inverse proportion to the skill of the men in shooting, or as has been said, "The better they shoot, the less they will get."

If each man is allowed to estimate the range for himself, the result will be a more or less increased dispersion, and the cone of fire which the company produces will be similar to that produced by a company composed of poor shots, using an elevation which is the mean of all the elevations assumed by the firers. Within a range

of 600 yards or less a correct determination of the range is relatively unimportant because of the flatness of the trajectory of the modern rifle, and the individual estimation of the range at this distance thus involves only the evil effects of a large dispersion similar to that produced by unskilled marksmen; at longer ranges the effect often is to increase the results over what would be expected from the same group controlled by an *untrained* officer, but always to diminish the result to be expected from the same men properly led.

CHAPTER II.

COMPUTATIONS.

In this chapter will be found gathered most of the mathematical principles involved in the remainder of the text. In order that these mathematical demonstrations may not confuse the student and divert his attention from the fire lessons which are the main purpose of the book, they are inserted here without comment and without deducing any but very general principles from the matter presented. The following chapters are thus freed from computations to such an extent that the student can concentrate his attention solely on the fire lesson, accepting as true the general statement of the mathematical results, or accepting them for the moment, he may later return to this chapter and prove their accuracy to his own satisfaction.

This arrangement involves unfortunate repetitions which might have been avoided by a more logical arrangement, but it is thought that for the majority of readers the advantages will outweigh the disadvantages of the chosen system.

PROBABILITY FACTORS.—All inquiry into the effect of fire is based upon a thorough understanding of the cone of dispersion of fire, for any conclusions reached as to the influence of this or that modifying factor on the efficacy of fire must be based either upon a long and carefully prepared series of experimental firing or upon the theory of probabilities as applied to a known dispersion. As these firings are impracticable, the student must fall back upon the theory; and, since the theoretical

results have been amply verified by actual firing he may be assured that in studying the theory of fire he has not made the mistake of departing from the things which are real and practical.

The dimensions of the shot group form the basis, then, of all study in fire effect, and it is important that the student should grasp the idea of what these dimensions are and why they are used.

When a detachment fires with a single point of aim, the shots will be found seemingly distributed over the target without any law, but more densely grouped about a central point called the CENTER OF IMPACT, or, as it is sometimes called, the MEAN, or MIDDLE POINT OF IMPACT. It will be observed, further, that the density of the shots decreases in all directions from the center of impact, gradually at first and then more rapidly until near the edges of the group there are but few hits. If the group is considered as having been made by a very great number of shots, then, by the theory of probabilities, we may deduce a law for the grouping, and determine the density of the group at any point or the per cent of hits to be expected in a strip of any assumed width.

If a horizontal line be drawn through the group at the center of impact it will divide the group into two equal parts and since the shots are symmetrically grouped about the center of impact, one-half of all hits will be above and one-half below this line. Now if two lines are drawn parallel and symmetrical to this line, and at a distance apart equal to the mean vertical dispersion, they will define a strip containing 50 per cent of all the hits, but if we draw another and similar strip one-half as wide as the mean vertical dispersion, it will not contain 25 per cent of the hits; but, because of the increas-

ing density as we near the center, it will contain 26.4 per cent of hits. Similarly a strip twice as wide as the mean vertical dispersion will not contain 100 per cent of hits, but 82.3 per cent because of the decrease in density. A strip $1\frac{1}{2}$ times as wide as the mean vertical dispersion will contain not 75, but 68.8 per cent, etc.

If the subdivision is continued a table can readily be formulated by reference to which we can see at a glance just what per cent of hits is to be expected in a strip of any given width, or the reverse of the problem, just how wide a strip must be in terms of the

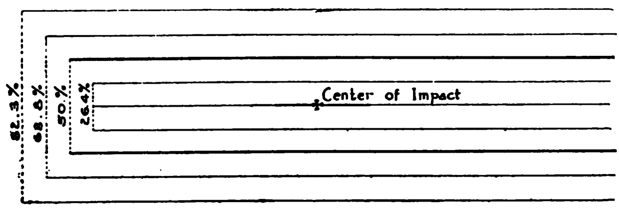


FIG. 17.

mean dispersion to contain a given per cent of hits. This ratio between the size of an assumed strip and the mean dispersion is called a PROBABILITY FACTOR, and being a ratio, it is, of course, applicable to any mean dispersion, vertical, lateral or longitudinal, and one of any dimension. Since only an infinite number would really correspond to 100 per cent of hits, the table is usually given for percentages between 1 and 99, or ratios between 0 and 4.0. Intermediate percentages or factors are found by interpolation.

TABLE I. PROBABILITY FACTORS.

Factor.	*Percent.*	*Factor.*	*Percent.*	*Factor.*	*Percent.*	*Factor.*	*Percent.*	*Factor.*	*Percent.*	*Factor.*	*Percent.*
0.02	1.1	0.52	27.4	1.02	50.9	1.52	69.5	2.02	82.7	2.55	91.5
0.04	2.2	0.54	28.4	1.04	51.7	1.54	70.1	2.04	83.1	2.60	92.1
0.06	3.2	0.56	29.4	1.06	52.5	1.56	70.7	2.06	83.5	2.65	92.6
0.08	4.3	0.58	30.4	1.08	53.4	1.58	71.3	2.08	83.9	2.70	93.1
0.10	5.4	0.60	31.4	1.10	54.2	1.60	71.9	2.10	84.3	2.75	93.6
0.12	6.5	0.62	32.4	1.12	55.0	1.62	72.6	2.12	84.7	2.80	94.1
0.14	7.5	0.64	33.4	1.14	55.8	1.64	73.1	2.14	85.1	2.85	94.5
0.16	8.6	0.66	34.4	1.16	56.6	1.66	73.7	2.16	85.5	2.90	95.0
0.18	9.7	0.68	35.4	1.18	57.4	1.68	74.3	2.18	85.9	2.95	95.3
0.20	10.7	0.70	36.3	1.20	58.2	1.70	74.9	2.20	86.2	3.00	95.7
0.22	11.8	0.72	37.3	1.22	58.9	1.72	75.4	2.22	86.6	3.05	96.0
0.24	12.9	0.74	38.2	1.24	59.7	1.74	75.9	2.24	86.9	3.10	96.4
0.26	13.9	0.76	39.2	1.26	60.5	1.76	76.5	2.26	87.3	3.15	96.6
0.28	15.0	0.78	40.1	1.28	61.2	1.78	77.0	2.28	87.6	3.20	96.9
0.30	16.0	0.80	41.1	1.30	61.9	1.80	77.5	2.30	87.9	3.25	97.2
0.32	17.1	0.82	42.0	1.32	62.7	1.82	78.0	2.32	88.2	3.30	97.4
0.34	18.1	0.84	42.9	1.34	63.4	1.84	78.5	2.34	88.5	3.35	97.6
0.36	19.2	0.86	43.8	1.36	64.1	1.86	79.0	2.36	88.9	3.40	97.8
0.38	20.2	0.88	44.7	1.38	64.8	1.88	79.5	2.38	89.2	3.45	98.0
0.40	21.3	0.90	45.6	1.40	65.5	1.90	80.0	2.40	89.5	3.50	98.2
0.42	22.3	0.92	46.5	1.42	66.2	1.92	80.5	2.42	89.7	3.60	98.5
0.44	23.3	0.94	47.4	1.44	66.9	1.94	80.9	2.44	90.0	3.70	98.7
0.46	24.4	0.96	48.3	1.46	67.5	1.96	81.4	2.46	90.3	3.80	98.9
0.48	25.4	0.98	49.1	1.48	68.2	1.98	81.8	2.48	90.6	3.90	99.2
0.50	26.4	1.00	50.0	1.50	68.8	2.00	82.3	2.50	90.8	4.00	99.3

Rule 1. *To find the percent of probable hits.*—Divide the width of the target in inches by the dispersion in inches, which will give the *probability factor,* then take corresponding percent from table. Example: Dispersion, 54 inches; target, 108 inches high; center of impact in center of target. Required the probable percent of hits.

The ratio of the target to the dispersion, i. e., the "probability factor," is $108 \div 54 = 2$. By reference to the table, a probability factor of $2 = 82.3$ percent.

Rule 2. *To find size of target* necessary to receive a given percent of hits. Take from the table the probability factor corresponding to the desired percent and multiply the dispersion by this factor. Example: Dispersion 54 inches, what size target is necessary with a properly placed center of impact to attain 82.3 percent of hits?

From table, 82.3 percent equals a probability factor of 2; 2x54 inches=108 inches, the target must, therefore, be 108 inches high.

Rule 3. *To find the range* at which a given percent will be attained against a target of given dimensions (center of impact centrally placed). From the table take the probability factor corresponding to the required percent. Divide the height of the target in inches by this factor, the quotient will be the dispersion necessary. In the table of dispersions find the corresponding range. Example: Target 108 inches high. At what range will average marksmen attain 82.3 percent of hits?

The probability factor corresponding to 82.3 percent of hits is 2. $108 \div 2 = 54$. The dispersion of 54 inches for average marksmen corresponds to a range of about 800 yards, which is the answer required.

There are other basic measures used by ballisticians, but the mean vertical and horizontal dispersions (50%) which is used throughout this text is the measure believed to be best suited to an inquiry into the effect of rifle fire.

The French frequently base their discussions on the "probable error" *(ecart probable)* which is equal to ½ the mean dispersion. We thus find *Lamiraux* and other French ballisticians referring to the mean vertical (50%) dispersion as the "Probable double vertical error."*

Another measure often encountered is a circle of such a size as to include 50 percent of all hits in the shot group, but as this does not lend itself readily to a study of rifle fire it is seldom used by students of small arms fire effect.

The mean vertical, horizontal and absolute *deviations* given in the accuracy tables published by the Ordnance Department, should not be confused with the mean vertical or horizontal *dispersion.* The figures as published are of value to the gun-maker and ordnance expert rather than to the student of rifle fire effect, as they serve chiefly for comparison of various arms and lots of ammunition. In determining these deviations, a great many targets are made under the most favorable conditions of light and air, the rifle being fired from a fixed rest placed in a firing house, the target is plotted and measured by using a camera placed in front of the target. Each series of 10 shots is computed separately

* The expression *"ecart probable"* is often translated as the "probable deviation." This is not strictly accurate, however, as *errors* are generally measured from the center of impact, *deviations* from the point of aim. As the practice is not uniform in this matter, a natural confusion arises when the term "probable error" or "probable deviation" is used.

on a printed target sheet, divided into a number of squares.

The point O is taken as a point of origin and the distance is measured vertically to each shot from the line O-Y, and horizontally from the line O-Z. In practice, and to avoid unnecessary measurements, an origin is assumed at a nearer point as at O′ adding to each

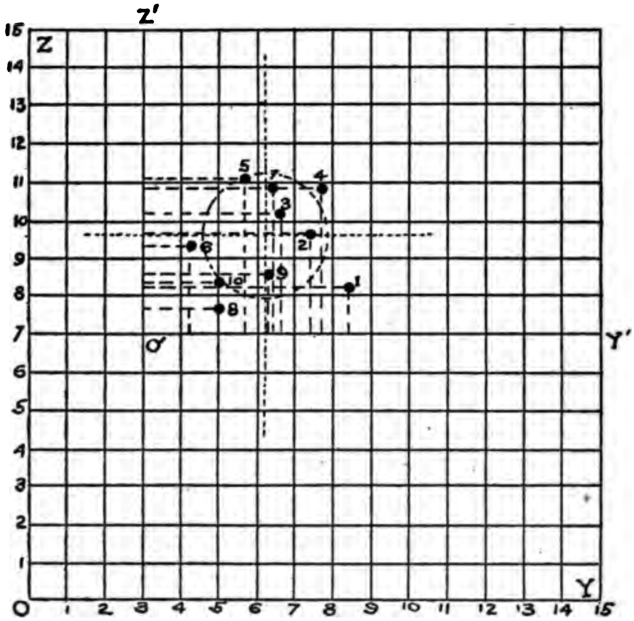


FIG. 18.

measurement the distance of this point from the origin. This is best illustrated by the figure (18) and its accompanying computations.

Beginning with shot number one, the opisometer by which the distances are measured is run vertically from the line O′-Y′ to the shot mark; it records 1.35 feet, to this reading is added 7 feet (the distance of the line

COMPUTATION OF TARGET SHOWN IN FIGURE 18.

	Vertical	Hori-zontal	
Shot No. 1	8.35	+8.45	
" " 2	+9.65	+7.45	
" " 3	+11.20	+6.55	
" " 4	+10.90	+7.70	
" " 5	+11.10	5.70	
" " 6	9.40	4.25	
" " 7	+10.90	+6.40	
" " 8	7.70	5.00	
" " 9	8.55	+6.30	
" " 10	8.45	5.00	
Footing	96.20	62.80	
Divided by 10	9.62	6.28	=Coördinates of center of impact
Footing of + coord.	53.75	42.85	
C of I × 5	48.10	37.67	
Dif. divided by 5	1.13	1.03	=Mean Deviations
Proof:			
Footing of — Coord	42.45	19.95	
C of I × 5	48.10	25.12	
Dif. divided by 5	1.13	1.03	=Mean Deviations
Square of M. V. Dev.	127.69		
Square of M. H. Dev.	106.09		
Sum of Squares	233.78		
Square Root of same	1.53		=Mean Absolute Deviation
Average of radial distances from C. I.	1.69		=Radius of circle of shots
1.13 × 1.69	1.91		=Mean Vert. Dispersion
1.03 × 1.69	1.74		=Mean Lat. Dispersion
$0.88 \sqrt{1.91 \times 1.74}$	1.82		=50% Radius

O′-Y′ from O-Y) and the vertical distance of shot No. 1 is recorded as 8.35. The sum of all the vertical measurements is 96.20 feet, and this divided by the number of shots (10) is 9.62 feet as being the distance of the center of impact above the line O-Y.

Similarly the horizontal distance of the center of impact is found to be 6.28 feet from the line O-Z. These distances are measured off and the position of the center of impact plotted.

It will be observed that 5 shots are above the center of impact and 5 below. Those above are designated as +, those below as — coördinates. To find the mean vertical deviation, the plus coördinates are added (53.75) and from this sum is taken the product of 5 times the distance of the center of impact (48.10). The result is the same as though the distance of the center of impact were first subtracted from the coördinate of shot 1, then of shot 2, etc., and the 5 remainders were added. The difference (5.65) divided by the number of shots (5) gives the mean vertical deviation as being 1.13.

As a proof of this result, the target is turned upside down and the operation is repeated with the minus coordinates, which give the same deviation (1.13). The horizontal deviation is found and proved in a similar manner to be 1.03. The mean absolute deviation is the resultant of these two factors and is found by extracting the square root of the sum of the squares of the factors. In this case it is 1.53.

It will be seen that the mean vertical deviation is simply the average distance of the shots above or below the center of impact. Now, by the theory of probabilities we can use this knowledge of the average position of a point to determine its probable position, that is to determine a point where an equal probability exists that any

shot will be found on one side or on the other of the point so found. It is not necessary to explain the whole theory, but it is sufficient to say that by multiplying the average by .8453 we will arrive at the probable deviation. Here, the *average* position of the shots above or below the center of impact was found to be 1.13 feet, and multiplying this by 0.8453 we find that the probable error will be .955 feet in the case considered. That is, that any given shot is exactly as likely to be found between the center of impact and the point 0.955 feet above as it is to be found at a greater distance above the center of impact.

This is the *écart probable* or "probable error" of the French. Above the center of impact, one-half of all shots will be found, and of these one-half will lie between the center of impact and the probable error and one-half above the probable error, that is to say, one-fourth of all the shots will not exceed a distance above the center of impact (in this case) of 0.955 feet, while another fourth will exceed that distance. As the shots are symmetrically grouped, the same conditions of grouping occur below the center of impact and 50 per cent of all hits will (in this case) lie in a strip 1.91 feet wide symmetrical to the center of impact—this is the MEAN VERTICAL DISPERSION of the group considered. *

* In determining the deviation, dispersion, etc., especially of the collective fire target, all abnormal shots are first discarded, so that if a target shows, say, 100 hits and two or three of those hits are evidently abnormal, the computation of the functions of the group would be based on the remaining 97 or 98 hits.

An abnormal hit is due to some unusual condition of arm, ammunition or man which causes the hit to fall so far outside of the group proper as evidently to be working under influences not present in the other shots. This is detected by applying the principles of the theory of probabilities to the questionable hit. If its position is a "probable" one it is accepted, if not it is rejected. These chance or abnormal shots occur with such regularity as to permit the determination of the number to be rejected directly from the number of hits on the target, and in practice we count the hits, subtract the most distant one—two, or three, etc., and proceed with the computation as explained.

The mean vertical and horizontal dispersion can, therefore, be determined from the accuracy tables by multiplying the given average or mean deviation by 1.69. The result will be the mean vertical dispersion for a vise-held rifle, however, and so is much smaller than can be expected from the field firing of troops.

In all problems which are based on the theory of probabilities, a very great number of shots are supposed to be considered. It is evident that if we fire but 10 shots they may be very erratically scattered on the target. but if we fire 100, the results attained will be much nearer the computed results in the matter of distribution; with 100,000 shots the grouping and expected per cent will almost exactly agree with the computed figures. When, in the following text, therefore, a certain per cent is predicted, it is assumed that the number of shots fired is great, and, further, it means that in one-half of all cases a greater result will be attained and in the other half a less result. The "probability" will be exactly equal. If a man has fired ten shots only and all of them have struck the bulls-eye, we would predict that the next string of 10 would also be bulls-eyes, the mathematical chance being $10 \div 10 = 1$. If in the second string he fired 10 shots and none of them struck the bulls-eye, the mathematical "chance" or probability would be $10 \div 20 = \frac{1}{2}$, and we would expect the next 10 shots to produce 5 bulls-eyes and 5 misses. We would say that the chance is equal for a miss or a bulls-eye on the first shot of the third string. Similarly here, we are dealing with a probability, that is when we predict a percentage of 5.4 we mean that an equal chance exists that the score will be greater or less than 5.4, or that the average of a number of scores will be 5.4.

The foregoing simple outline of the law of chance is made that the student may not be misled into assuming that a computed result is incorrect because it is not borne out by firing one score, and that he may appreciate the difference between an average error and a probable error. Taking an example in the matter of estimating distances; a company of 100 men estimates the distance to an object and records the several independent estimates, divides the sum of the errors by the number of estimates and records an average error of 200 yards in a true distance of 1,000, or 20 per cent. This is the average error, but not the *probable* error, for in attaining this average many of the men made a much smaller error, the majority of the company, in fact, making less than 200 yards error, but because of a few very poor guesses, the average was pulled down to 20 percent. Taking any man at random from the company, one would say that his error in estimating 1,000 yards would be less than 200 yards, and that his error would probably be 20% × .8453, or 16.9%, for half of all the 100 men estimated the distance with an error of less than 169 yards, and the other half made more than 169 yards error, so that the chances of a greater and of a less error are exactly equal and in a very great number of cases there will be just as many estimates over this error as there are less than it.

INFLUENCES AFFECTING THE DISPERSION.

It has already been pointed out that whenever a number of shots are fired at a target the shots are symmetrically grouped about the center of impact and the causes of the dispersion have been shown to be the variations in initial velocity, density of the air and differences in the angle of departure or of elevation. As these devi-

ating influences act longer on the bullet at the long ranges than at the short, it follows that the resulting dispersion will be greater at the long ranges.

Considering the causes in detail, it is found that the dispersion caused by the variations in the initial velocity, air resistance, wind, etc., increase much more rapidly than the range, while the influence of the errors in the angle of elevation ("angular errors") increases with the range. For example, a change of 1 minute in the angle of elevation due to faulty aiming, and a variation of 40 feet in the initial velocity (the initial velocity of any lot of cartridges may vary 40 feet from that stamped on the bandolier) will produce a change in the point of impact of —— inches, or a change in the range of ——— yards at ——— yards.

Range.	*Angular error of 1′ causes.*		*Change of 40 ft. in initial velocity causes*	
	Vert. change inches	*Hor. change Yards*	*Vert. change inches*	*Hor. change Yards*
500	5.2	24.0	1.9	4.0
1000	10.4	11.0	8.6	4.6
1500	15 6	6.9	23.2	5.1
2000	20.8	4.9	53.5	6.2

An error of only 1 minute in sighting is smaller than that to be expected from average marksmen and can very easily be made by even good marksmen with good eyesight. The cartridges taken from the back pouches where they have been warmed by the sun will vary in initial velocity as much as 40 feet from those taken from the front pouches, which have beeen chilled by contact with the ground. A cartridge allowed to remain in the chamber of the rifle during a pause in the firing will vary as much as 100 feet or more in the velocity, etc.

Examining the table, it is seen that the effect of the angular error of $1'$ is 4 times as great at 2,000 yards as at 500, while the effect of even as small a variation as 40 feet in the velocity is 30 times as great at 2,000 yards as at 500. The human and avoidable errors are those of sighting, etc. (angular), and these are most important at the short ranges where the effect of the unavoidable errors is relatively unimportant. The reverse condition is observed at the long ranges where the dispersion is affected to a great extent by the unavoidable errors, and is influenced only in a minor degree by the human errors of sighting. The reason for the difficulty of making a good score at the long ranges then is not one of sighting, but of judging the character and amount of the meteorological factors, since the same correctness of sighting which insured a high score at 300 yards will not insure—and indeed will scarcely affect—the score at 2,000.

So soon as the dispersion caused by the meteorological conditions becomes greater than the dispersion caused by angular errors in sighting and holding, it is evident that intelligent instruction cannot be given the soldier in aiming, for the deviation of the shots from the center of impact are due, then, not to the faults of the firer, but to causes beyond his control. As it is only at the short ranges that the angular errors are the most potent, it is clear that instruction in aiming cannot profitably be extended beyond about 500 yards, where computation of the influence of the two classes of errors shows the dispersion to be about the same.

At a range of 500 yards, a miss made by the soldier is as likely to have been the result of meteorological conditions as to have been due to faulty aiming. Below

this range, and in an increasing measure, the soldier's errors in aiming can be pointed out to him and corrected. It is for this reason that all governments except our own have reduced the ranges at which individual instruction is given at bulls-eye targets or targets containing a bulls-eye to about 400 meters (440 yards), although group firing is conducted in all armies at greater ranges—the French, for example, fire up to 3,000 meters (3,300 yards) in their field practice.

It is important that one should understand the law by which the dispersion increases with the range under the influence of several dispersing factors acting together.

Let us suppose, using the figures just deduced, that a rifle is absolutely perfect, so that every shot fired would hit the spot aimed at, and that in aiming a mean error of 1 minute was made, this would cause a mean dispersion of 10.4 inches at 1,000 yards. Now, if we suppose that all the errors of aiming are eliminated, as well as all errors of the rifle, except that of initial velocity, which produces a mean variation of 40 feet per second, then the dispersion from this source would be 8.6 inches at 1,000 yards.

If these two sources of error acted at the same time, the combined effect would be greater than either one and less than the sum of the two, for acting under one the bullet may tend to go high, and the other may at that shot be pulling it down. The amount of the deviation caused by the combined forces would be the square root of the sum of their squares according to the formula of the French ballistician Didion, the formula being the expression of what is called "Didion's Law."

Graphically, and considering only these two sources of dispersion, the combined dispersion would be the

hypothenuse of a right angled triangle whose sides are respectively 10.4 and 8.6 units long. (Fig. 18a.) The combined dispersion would be 13.5 inches.

If the sides of the triangle are very unequal, the hypothenuse would vary only a little from the longer side, hence the extent of the dispersion depends principally on the extent of the dispersion arising from the greatest source of error.

Because of unavoidable differences in the manufacture of rifles, it is found that in a lot of rifles picked at random, some shoot high, some low, some to one side and some to the other, though each rifle has about its own center of impact a group of approximately the

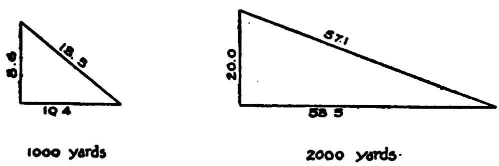


FIG. 18-a

standard dimensions. A shot group made by a number of these rifles, each carefully and properly sighted and fired from a rest, would evidently be larger than that of any one of the single groups, because of this dispersion of the centers of impact. If we assume, in order to illustrate the law and not because experiments have shown the statement to be exact, that the dispersion of the centers of impact is as great as the dispersion of the individual rifle group, then the combined effect—as a result of the armament—would be 21.2 inches at 1,000 yards. For the mean vertical *deviation* at 1,000 yards (from the Manual) is 8.9 inches, the mean vertical *dispersion* would, therefore, be $1.69 \times 8.9 = 15$ inches and

assuming 15 inches as the dispersion of the centers of impact—the combined effect would be

$$\sqrt{15^2 + 15^2} = 21.2 \text{ inches}$$

If we now assume a mean error of 5 minutes in aiming (a mean variation of 0.035 inches in the amount of the front sight seen, for example), this would cause 52.0 inches dispersion at 1,000 yards, and combining these last two the dispersion would be:

$$\sqrt{21.2^2 + 52^2} = 56.0 \text{ inches}$$

There are many other factors which enter into the complete problem, but enough has been shown to illustrate the law.

In target firing (individual practice), the marksman must take into account variations from the normal of the initial velocity, density of the air, temperature, etc., but in field firing these influences are relatively small enough under anything but very exceptional conditions to warrant the fire director in roughly estimating them in yards, or—at the shorter ranges—ignoring them altogether.

It is usual to graduate the sights of a rifle for the average meteorological conditions of the country, and this is practicable in those European countries where the range in temperature is comparatively limited; in a country such as ours, however, where the temperature varies from —40° F. to + 110° F., it is evident that the figures marked on the rear sight leaf willl seldom be correct.*

* Since much of our service is in the Tropics, where the range of the temperature and barometric pressure is relatively small, it might be wise to follow the example of Holland and provide special rear sights for our colonial troops.

The change above or below the normal sighting of the rifle which will be caused by variations in any of the foregoing particulars (air, velocity, wind, etc.), are—within the limits of any probable temperature or pressure—less than the mean longitudinal dispersion for average marksmen, and only a little greater at ranges beyond 1,500 yards than that of good marksmen with whom we will have very little to do in war. Since an officer cannot know the exact meteorological conditions existing on the battlefield nor the changes in elevation produced thereby, it is evident that only by personal practice under varying atmospheric conditions and at the longer ranges can he learn to form a trustworthy estimate of the addition or reduction necessary in any given case. He may be assured, however, that the errors in elevation introduced by the meteorological conditions will always be less in extent than those introduced by the error in the determination of the range to the target. For example, the change in temperature of 10° Fahr. alters the range by about 1.4 per cent. of the distance, so that a temperature change of 10° would alter the range 21 yards at a distance of 1,500 yards, at which range the mean longitudinal dispersion of good marksmen is 36 yards, and the probable error in estimating the range is 188 yards.

SIZE OF THE DISPERSION.

For the purpose of comparing the relative importance of the factors which modify the probable effect of fire in detachment field firing, any reasonable dispersion will serve so long as it is not grossly different from what may really be expected of marksmen of the class under discussion. It is unfortunate that the United States Army has never made the necessary experiments

to determine the dispersions of the several classes of marksmen which are recognized.

The most reliable data on the subject of dispersions is that obtained by a very extensive series of firings made at the German School of Musketry at Spandau under the direction of very careful and painstaking scientists. The ballistic value of the rifle with which this firing was done approaches very closely to our own, and the marksmanship of the firers is probably close enough to what we may expect from our own men to warrant their use when properly modified.

A graphical idea of the dispersions found in the various foreign armies (Fig. 19) gives a fair conception of the relative value attached to the well-known marksmanship of the American soldier, as compared with several foreign armies. The several dispersions are for a range of 500 yards, and the target "B" shown is drawn to same scale as the dispersions. The ellipse covers 19.64 per cent. of all the hits.*

For convenience of discussion three classes of marksmen will be considered—good, average and poor—and the dispersion of each as given in table II is based on the experimental firing referred to and other data, all properly modified according to computed influences not present in the experimental firing, such as differences in the rifle and ammunition. For the poor marksmen class an arbitrary assumption was made that their dispersion is twice that of average marksmen. Such American experimental firing as has been done indicates that these

* The major axis of the ellipse or its long diameter is made equal to the mean vertical dispersion, the minor axis or short diameter is made equal to the mean lateral dispersion. The rectangle determined by these diameters contains 25% of all hits and the ellipse within the rectangle 0.7854 of that amount, or 19.64%.

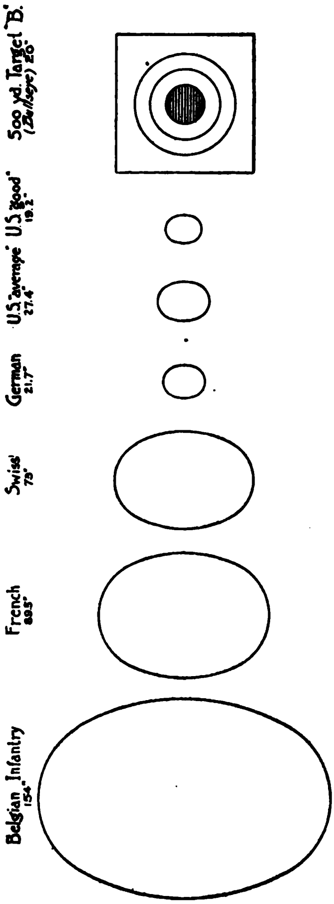

Belgian Infantry
154"
French
89.5"
Swiss
75"
German
21.7"
U.S. 'average'
27.4"
U.S. 'good'
19.2"
500 yd. Target 'B'
(Bullseye) 20"

FIG. 19

TABLE II. DISPERSIONS IN GROUP FIRING.

Range.	(50%) Mean Vertical Dispersion.			(50%) Mean Lateral Dispersion.			(50%) Mean Longitudinal Dispersion.		
Yards.	*Good Inches.*	*Average Inches.*	*Poor Inches.*	*Good Inches.*	*Average Inches.*	*Poor Inches.*	*Good Yards.*	*Average Yards.*	*Poor Yards.*
100	2.5	3.00	6.00	2.5	3.00	6.00			
200	5.3	7.75	15.50	5.3	7.75	15.50			
300	8.9	12.50	25.00	6.9	9.7	19.4			
400	13.20	18.50	37.00	10.4	14.6	29.2	86.50	121.00	242.00
500	19.20	27.40	54.80	14.20	20.20	40.40	84.50	120.00	240 00
600	25.00	36.20	72.40	17.80	25.80	51.60	78.50	114.00	228 00
700	29.70	45.20	90.40	20.60	31.40	62 80	68.50	105.00	210.00
800	35.00	54.30	108.60	24.10	37.50	75.00	61.50	95.00	190.00
900	40.50	63.40	126.80	27.80	43.50	87.00	55.00	86.00	172.00
1,000	46.50	72.50	145 00	31.60	49.50	99.00	50.00	77.00	154.00
1,100	53.00	81.70	163.40	36.00	55.50	111.00	45.50	70.00	140.00
1,200	60.50	91.00	182.00	41.00	61.50	123.00	43.50	65.00	130.00
1,300	67.70	101.30	202.60	45.00	67.40	134.80	40.50	60.50	121.00
1,400	74.50	111.20	222.40	49.50	73.50	147.00	38.00	57.00	114.00
1,500	82.60	121.40	242.80	54.00	79.50	159.00	36 00	53.50	107.00
1.600	91.00	133.00	266.00	58.50	85.50	171.00	34.80	50.50	101.00
1,700	100.50	146.20	292.40	65.00	94.00	188.00	33 80	49.20	98.40
1,800	110.50	160.00	320.00	71.00	103.00	206.00	33.40	47.30	94.60
1,900	129.50	176.00	352.00	82.00	113.00	226.00	34.20	46.20	92.40
2,000	141.00	192.00	384.00	90.50	122.00	244.00	33.00	45.00	90.00
2,100	155.50	212.00	424.00	102.00	137.00	274.00	32.60	44.50	89.00
2,200	171.50	234.00	468.00	112.00	153.00	306.00	32.40	44.00	88.00

dispersions will be confirmed when a proper amount of firing shall have established the true dispersions. Considered and compared as individual scores at 500 yards (no marking between shots, and the average for the whole company being taken) these dispersions would give:

For good marksmen, a score of 35.3, or a percentage of 70.6.

For average marksmen, a score of 29.2 or a percentage of 58.4.

For poor marksmen, a score of 23.0, or a percentage of 46.0.

COMPUTATION OF EXPECTED HITS.

With the assistance of the foregoing tables it is a simple matter to foretell the probable number of hits which will result upon any given target, or to determine the related questions as to the number of rounds necessary to effect a given number of hits, to compute the length of time necessary at slow fire to fire those rounds and at rapid fire; to predict the number of hits when the point of impact coincides with the point of aim and when it does not, and so to study the effect in terms of hits which the various modifying factors such as an incorrect estimate of the elevation, etc., will produce.

EXAMPLE 1. Taking the simple case of the wall target and assuming that a board fence three feet high has been constructed of infinite length with a horizontal aiming line drawn through its center. One hundred men from a prone position, on a favorable day, are firing at 500 yards range (correct elevation), each man firing straight to his front.

By reference to Table II we see that the mean vertical dispersion of average marksmen at 500 yards is 27.4 inches. The ratio between this dispersion and the height of the target, i. e. the "probability factor," is therefore $36 \div 27.4$, or 1.30. From Table I we find that the percentage corresponding to a factor of 1.30 is 62, we would therefore "expect" 62 hits out of each 100 shots and 38 misses; 19 going over and 19 going under the target. This does not mean that 62 hits would in all cases be obtained, but that it is probable that 62 per cent. of hits will be obtained if a great number of shots are fired, and if

the center of impact lies exactly in the center of the target.

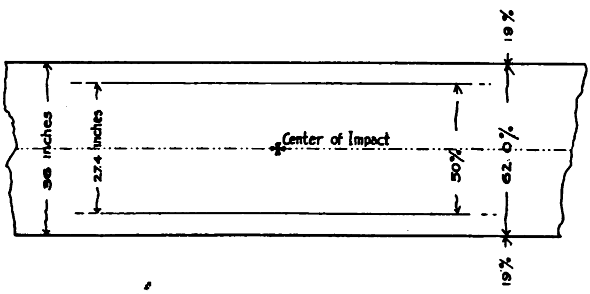


FIG. 20

With good shots, the dispersion would be 19.20 (from Table II), and the Probability Factor 1.87 ($36 \div 19.20 = 1.87$), which corresponds to a percentage of 79.25 (Table I). We would expect, therefore, 79.25 per cent. from this class of marksmen, while poor marksmen would be expected to produce 34 per cent. ($36 \div 54.8 = 0.65 = 34$ per cent).

EXAMPLE 2. How high must a very long target be in order to receive 60 per cent. of hits at 1,000 yards?

The probability factor corresponding to 60 per cent. is 1.25, the target must be 1.25 times as high as the mean vertical dispersion at 1,000 yards, which, for average marksmen, is 72.50 inches. $1.25 \times 72.5 = 90.63$ inches, or a little over 7.5 feet.

EXAMPLE 3. At what range is it probable that a very long target 6 feet high will receive 80 per cent. of all shots fired at it?

Six feet $= 72$ inches. The probability factor corresponding to 80 per cent is 1.90. The mean vertical dispersion must not be greater than $72 \div 1.90 = 38$ inches.

For average marksmen, this is the dispersion at 623 yards (from Table II by interpolation).

In the foregoing examples it was assumed that the target was long enough to receive all the shots. When the target is narrow, but very high, a similar computation can be made, using the mean lateral dispersion, thus:

EXAMPLE 4. What per cent. of hits will a target 36 inches wide receive at 1,000 yards, provided that it is so high that no shot can go over it?

At 1,000 yards the mean lateral dispersion is 49.5 inches (Table II), the probability factor, therefore, is $36 \div 49.5 = 0.73$, which corresponds to 37.5 percent.

In the first three examples a target was assumed of such length that no shot could miss except high or low; in the fourth example a similar assumption excluded consideration of the high or low misses. Where the target is limited in both directions and the center of impact lies in the center of the target we must go one step farther and, obtaining each per cent. separately, multiply them together to obtain the probable result. The reason for this will be understood when it is remembered that the shots are symmetrically arranged about the center of impact.

Considering, for example, the rectangle formed by the intersection of the two 50% strips. (n o p g Fig. 21.)

Since the mean vertical dispersion strip (e f g h) contains 50 per cent. of all the hits on the target and the shots are arranged symmetrically it will contain 50 per cent. of the hits found in any vertical strip which is symmetrical to the center of impact. The mean lateral dispersion strip (i k l m) is such a strip, and is known to contain 50 per cent of all hits, hence in that portion of this vertical strip which is included in the horizontal strip will

be found 50% of 50%, or 25% of the total number of hits on the target.

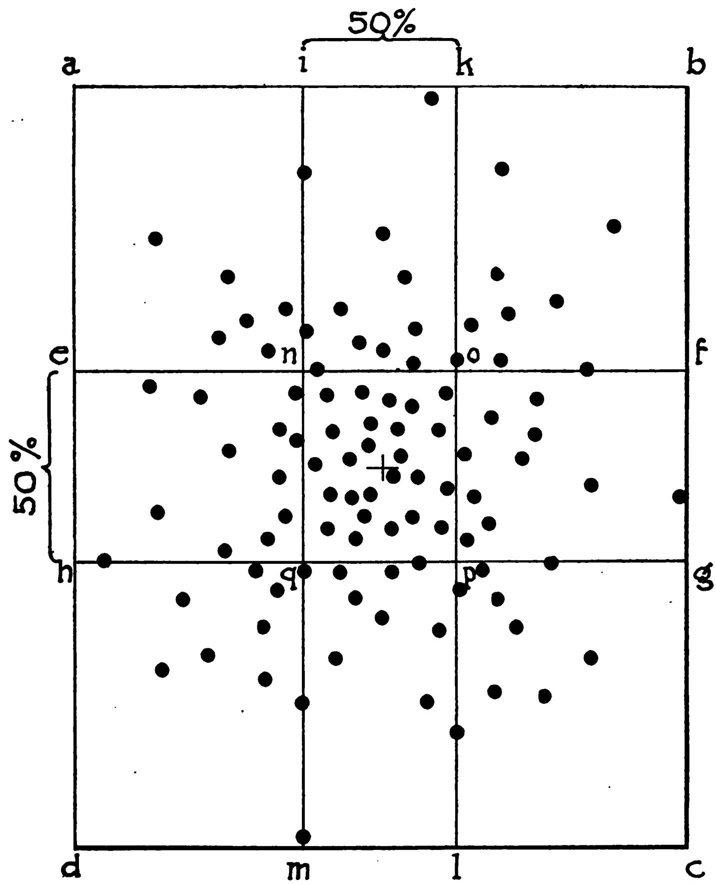


FIG. 21.

When considering a target of fixed dimensions, therefore, the rule is first to determine the percent to be expected, as in examples 1, 2 and 3, presuming the target to have infinite length; then to compute the probable per-

centage on the assumption that the target has infinite height. The product of these partial results will be the percentage to be expected in the considered target, *if the center of impact lies exactly in the center of the target.*

EXAMPLE 5. What per cent. of hits is probable on a target 6 feet square at 1,000 yards with average marksmen, the center of impact being in the center of the target?

The mean vertical dispersion is 72.5 inches, the probability factor is $72 \div 72.50 = .99 = 49.5$ per cent.

The mean lateral dispersion is 49.5 inches, the probability factor is $72 \div 49.50 = 1.45 = 67$ per cent.

The expected percentage on the target 6 feet square is, therefore, $67 \times 49.5 = 33.2$ per cent

In the foregoing example it was assumed that the center of impact always lay in the center of the target. This is the exception rather than the rule in detachment firing, hence in considering that class of firing it becomes necessary to compute the influence of a misplacement of the center of impact. The computation may be illustrated by three typical cases.

1. The center of impact lies on the (upper) edge of the target.

2. The center of impact lies on the target, but not at the center.

3. The center of impact lies outside of the target.

(Firing as before on a board fence 3 feet high, of infinite length with average marksmen at 500 yards, the mean vertical dispersion being 27.4 inches.)

EXAMPLE 6. The center of impact is at the upper edge of the target.

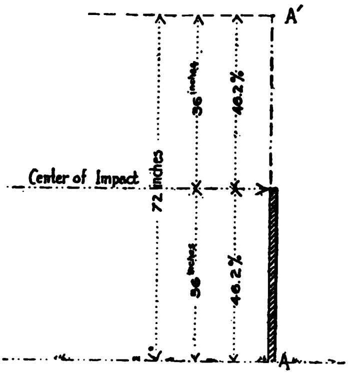


FIG. 22

The center of impact is 36 inches above the center of the target by the statement of the problem. Since the shots are symmetrically grouped about the center of impact, there will be as many shots found in a space 36 inches above the center of impact as below, hence if we determine the probable number of hits in a strip 72.0 inches wide (A A' Fig. 22), with the center of impact in the center of the strip, we will reach a result just twice as great as should be expected on the target.

The dispersion is 27.4 inches, the probability factor is $72 \div 27.4 = 2.63 = 92.3$ per cent. As stated, one-half of this expected 92.3 per cent. lies above and one-half below the center of impact, so that on the target 46.2 per cent. of all shots will probably be found.

The Center of Impact is 10 Inches Above the Middle of the Target.

Example 7. The center of impact is 10 inches above the center of the target, or 28 inches above the bottom edge. Considering an equal space above the center of impact, or a total strip of 56 inches. The dispersion being 27.4 inches and the "strip" 56 inches, the probability factor would be $56 \div 27.4 = 2.05 = 83.4$ per cent., and one-half of this per cent., or 41.7 per cent. would lie below the center of impacct.

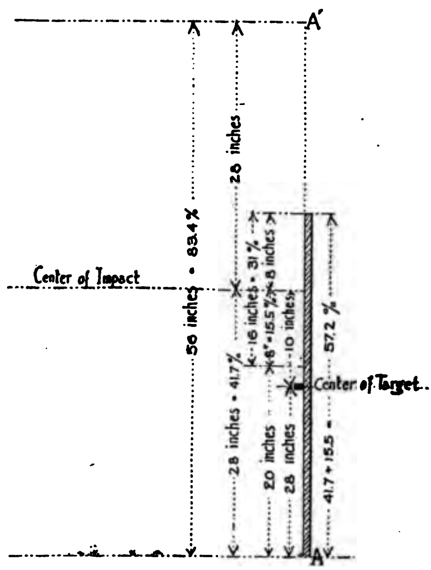


Fig. 23.

This leaves a strip of the target 8 inches wide lying above the center of impact still to compute. Doubling this 8 inches gives a strip 16 inches wide to consider, and this 16-inch strip would contain $16 \div 27.4 = .59 = 31$ per cent., one-half of which, or 15.5 per cent., lies above and the other half below the center of impact. In the whole target, therefore, we would expect 57.2 per cent., 41.7 per cent. below the center of impact and 15.5 per cent above.

EXAMPLE 8. The center of impact is 38 inches above the center of the target.

Since the target is 36 inches high, 18 inches of its height lies above the center of the target, and since by the statement of the problem the center of impact is 38 inches above the center of the target, it evidently is 20 inches above the top of the target ($38 - 18 = 20.$) From the center of impact to the bottom of

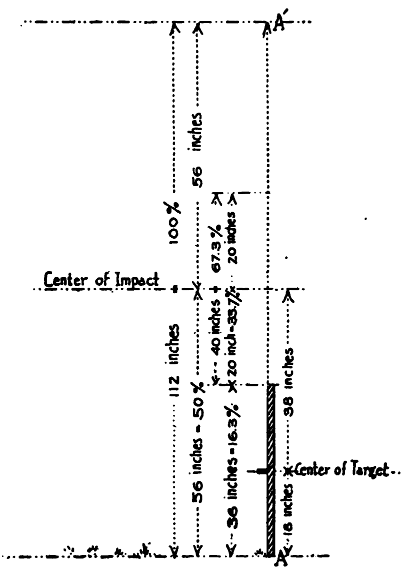


FIG. 24

the target is 38 inches plus 18 inches, or 56 inches. Doubling this and computing the expected percentage in the strip A-A', 112 inches wide, the probability factor is $112 \div 27.4 = 4.1$. This factor really corresponds to 99.43 per cent., but it is usual to consider any factor of 4 or more as corresponding to 100 per cent. All of the shots would be found, therefore, in the space A-A', and 50 per cent. in the space between the center of impact and the bottom of the target. This includes 20 inches of clear space above the target, and doubling this and determining the per cent. of hits to be expected in the 40-inch strip, then in the 20-inch space, and subtracting the amount thus found from the 50 per cent. expected below the center of impact will leave 16.3 per cent. as the probable per cent. of hits in the target. The

computation is as follows: In the strip A-A′ = 112 inches, 100 per cent. (by observation, since 4 times the mean vertical dispersion equals 109.6, which being less than the size of the strip—112—shows a factor of over 4), and in the lower half of this strip 50 per cent. would be anticipated. In the clear space between the center of impact and the top of the target; $2 \times 20 = 40$. $40 \div 27.4 = 1.46 = 67.3$ per cent. $67.3 \div 2 = 33.7$ per cent. $50 - 33.7 = 16.3$ per cent.

A similar series of examples would show the effect of a lateral displacement.

EXAMPLE 9. On a very high target, 36 inches wide, what would be the effect of two points of windage erroneously taken?

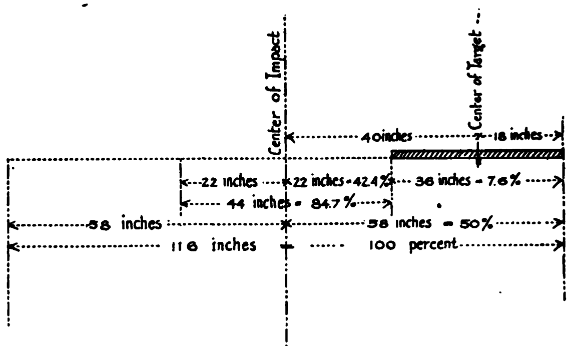


FIG. 25.

From the table of windage effects, at 500 yards, 1 point is found to move the center of impact 20 inches; 2 points would move it 40 inches, or it would lie 22 inches outside the target. The mean lateral dispersion at 500 yards is 20.7. The probability factor is $116 \div 20.7 = 4+ = 100$ per cent. $100 \div 2 = 50$ per cent. The probability factor of the clear space is 2×22

= 44 inches. 44 ÷ 20.7 = 2.12 = 84.7. 84.7 ÷ 2 = 42.4 per cent. 50 — 42.4 = 7.6 per cent.

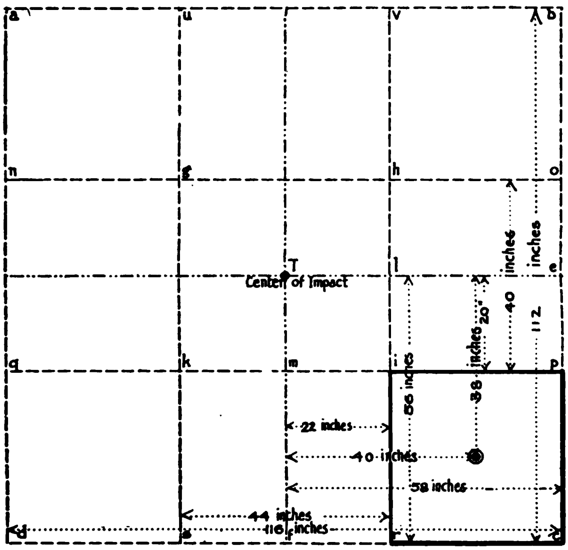


FIG. 26.

If we combine two errors so as to displace the center of impact high and to the right or left, the problem becomes one of the probability of hitting a part of a plane figure and is a little more complicated.

Combining the errors of examples 3 and 4, the center of impact would lie at T (Fig. 26), 38 inches above the center of impact and 40 inches to one side.

All of the shots fired will lie in the rectangle a-b-c-d, since both of its dimensions exceed 4 times the dispersion, and since the shots are symmetrically grouped, 1/4 of the whole number of shots will lie in the rectangle T-e-c-f, or 25 per cent. In the rectangle g-h-i-k, 44 inches wide and 40 inches high will be found 57 per cent.

$40 \div 27.4 = 1.46 = 67.3$; $44 \div 20.7 = 2.12 = 84.7$; $84.7 \times 67.3 = 57$ per cent), and one-fourth of this per cent., or 14.2 in the rectangle T-l-i-m. In the rectangle n-o-p-q, 40 inches high and wide enough to receive all shots, would lie 67.3 per cent of hits, 16.9 per cent., or one-fourth, being in the rectangle T-e-p-m. This rectangle includes T-l-i-m just computed at 14.2 per cent., hence in the remaining rectangle l-e-p-i will be found $16.9 - 14.2 = 2.7$ per cent. In a similar manner the rectangle m-i-r-f is computed. The rectangle r-s-u-v contains $44 \div 20.7 = 2.12 = 84.7$ per cent., and the rectangle T-l-r-f one-fourth of that amount, or 21.2 per cent. As before, this includes the rectangle T-l-i-m with its 14.2 per cent., hence the rectangle m-i-r-f contains $21.2 - 14.2 = 7.00$ per cent.

Combining the three rectangles gives 23.9 per cent. as being in the figure T-e-p-i-r-f ($14.2 + 2.7 + 7.0 = 23.9$). Subtracting this amount from the 25 per cent. known to be in the rectangle T-e-c-f gives 1.1 per cent. as the probable number of hits in the target i-p-c-r.

FIGURE TARGETS.

Hitherto we have discussed the effect of the rifle against target walls (solid surfaces) and have shown that the effect depends upon the ratio of size of dispersion and of the target wall, and upon the position of the center of impact with reference to the center of the target. In combat we will not have solid targets, but lines of varying density made up of individual men with spaces between. If a line of figure targets (silhouettes) be placed against the solid target wall which we have been considering, then only a portion of the hits will be found in the figures while the others will lie in the space between and around the figures. The number of hits in the

figures will bear the same proportion to the total number of hits on the whole target as the area of the figures bears to the total area of the target wall.

In example 5 (page 70) we found that against the target 6 feet square under the conditions stated in that problem 33.2 per cent of hits was to be expected. If one silhouette (target H) were pasted in the center of the 6-foot target we would expect 5.34 per cent of the hifs to lie in the figure and 27.86 per cent to lie in the space surrounding the figure, for the area of the target is 5184 inches, and of the silhouette 833 square inches, and 5184 : 833 : : 33.2 : 5.34. In such computations for the comparison of areas the area of a rectangle is computed which will just contain the figure, and as each man of the firing detachment fires straight to his front, the lateral dispersion does not enter into the question, as the target is considered as long enough to include the entire lateral dispersion of the shot group. This introduces a slight error at the ends of the line of targets, but this error is negligible. Suppose that on a target wall 68 inches high, standing silhouettes are placed so as to represent infantry in close order (single rank). Each figure would occupy a width of 26 inches (Inf. D. R.), and a height of 68 inches (height of target H), and of each 1768 square inches ($26'' \times 68'' = 1768''$), 833 square inches would be vulnerable (area of target H = 833 square inches).

The hits to be expected on the figure would be to the hits expected in the rectangle as 833 is to 1768, or as 0.472 is to 1. With average marksmen firing at 800 yards against this target, we would expect on the wall target 60 per cent of hits (target 68 inches; mean vertical dispersion 54.30 inches; probability factor $68 \div 54.3$

$= 1.25 = 60$ per cent), and on the figures 28.3 per cent of hits ($60 \times 0.472 = 28.3$: or $1768 : 833 :: 60 : 28.3$). This is a matter of great importance in judging field firing of troops, for if the figures are placed at one man per yard of front, there would be one figure in each strip of 36 times 68 inches, or 2448 square inches, and the per cent of hits on the figures would be $60 \times 833 \div 2448 = 20.5$. While with the figures at two pace intervals, there would be one figure in each rectangle of $84 \times 68 = 5712$ square inches (2 paces $= 60$ inches, one figure $= 24$ inches; $60 + 24 = 84$), and the per cent of hits on the figures would be $60 \times 833 \div 57.12 = 14.6$.

A review of these figures shows that with uniformally good shooting very different results are obtained according to the target, for, although only half as many hits on the figures are made in the last case as in the first, the shooting is exactly as good.

In the preceding examples the vulnerable area of the figure was taken as 833 square inches. In nearly all armies the silhouettes used in target practice bear very little resemblance either in size or color to probable battle targets, although a few of the Continental governments have adopted "invisible" colors, and some, notably the German, have reduced the size of their targets to more nearly the average human size.

If we accept the photogrammetric measurements of the Italian investigators as representing the true vulner-

able surface of a man and of a horse, we shall have the following figures as a basis for computing firings on battle targets:

	Sq. in.	American Target.	German Target.	
Inf. soldier standing, from front	740	68×24; 833 sq. in.	55×20 "standing"	; 800 sq. in.
Inf. soldier standing, from side	434	....	....	; 465 sq. in.
Inf. soldier kneeling, from front	530	42×26; 655 sq. in.	31×20 "kneeling"	; 464 sq. in.
Inf. soldier lying down, from front	250	22×26; 337 sq. in.	20×20 "breast"	; 283 sq. in.
Inf. soldier in trench, front	185	....	12×20 "head"	; 159 sq. in.
Horse only, from front	1295	....	....	; 1240 sq. in.
Horse only, from side	2465			
Horse and rider, from front	1750	....	....	; 1860 sq. in.
Horse and rider, from side	2795			

In the subsequent examples these (photogrammet-

ric) areas will be used as a basis, though they will not in every case be absolutely true, as for instance, where a man lying down is considered, since the vulnerable surface of such a target, depending upon the slope of the ground and the angle of fall of the bullets, will present a vulnerable surface in excess of that here given and in a varying degree. For example, the vulnerable area of a man lying down in the open is 250 square inches, and if the trajectory were horizontal and the man on level ground, this would represent his theoretical and practical vulnerable surface. But if the trajectory makes an angle with the horizontal, then the vulnerable surface will be increased. Thus (Fig. 27), if m = the vulnerable area on level ground, and with horizontal trajectory = 250

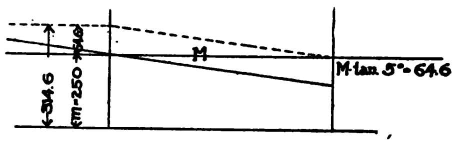


FIG. 27

square inches, and M = the vulnerable area of a standing man = 740 square inches, then the projection of M in a vertical plane, when the angle of fall is 5 degrees, will be $M_v = M \tan 5° = 740 \tan 5° = 64.6$ square inches, and this added to m = 314.6 square inches. So that, if the angle of fall be 2°, as it is at about 1000 yards, and if the figure is lying on an upward slope of 3° its vulnerable area will be one-fourth greater than it would be on a plain and at a short range where the trajectory is practically horizontal.

The data thus obtained is, of course, not precise, but it affords a means of comparing the vulnerability of men in different positions. It shows, for example, that

the vulnerable surface of a standing man is not three times as great as one lying down, but only 2.35 times as great (in the case worked out), and that a mere comparison of the tabular areas (250:740, or 1:3) would be misleading.

DENSITIES.

It sometimes becomes necessary, especially when dealing with the longitudinal dispersion of the horizontal shot group, to know the probability of hitting any particular point within the dispersion, that is, to know the

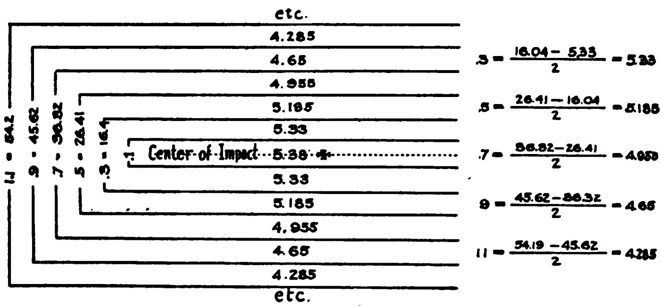


FIG. 28.

density of the shot group at a point a given distance from the center of impact.

On the assumption that the density of hits in a horizontal strip is uniform within that strip, we can compute the probable density at any given point within the whole shot group.*

From Table 1 we find that 5.4 per cent of hits are expected in a strip 0.1 as wide as the mean dispersion (the probability factor 0.1 corresponding to 5.4—or more exactly 5.38). Similarly we find that the prob-

***Based on the table compiled by Wuich, in his "Theory of Probabilities."**

ability factor 0.3 corresponds to 16.0 per cent—(exactly 16.04), and so on throughout the table. If, then, we draw a horizontal strip one-tenth as wide as the mean dispersion and so placed that its center will coincide with the center of impact, we would expect in this strip, the distance of whose center from the center of impact is 0.0, 5.38 hits out of every 100 shots fired. Drawing two strips of the same width (0.1 of the mean dispersion) on each side of, and symmetrical with the center strip, the centers of these strips would be 0.1 of the mean dispersion distant from the center of impact, and since the three strips contain 16.04 hits on a basis of 100 hits in the group, and 5.38 of these lie in the center strip, then $(16.04 - 5.38 =)$ 10.66 hits would be found in the other two strips, half in each, or 5.33 because of the symmetrical arrangement of the hits. In each of the next pair of the strips, whose centers are 0.2 of the mean dispersion distant from the center of impact, would lie 5.185 hits, for the five included strips have a width .50 as wide as the mean dispersion which corresponds (by Table 1) to 26.4 per cent—exactly 26.41. In the three center strips we found 16.04, hence in each of the strips now considered would lie $(26.41 - 16.04) \div 2 = 5.185$. Continuing this process throughout the whole group and tabulating the results gives:

N	P	N	P	N	P
0.0	5.380	1.0	2.165	2.0	0.140
0.1	5.330	1.1	1.795	2.1	0.100
0.2	5.185	1.2	1.450	2.2	0.065
0.3	4.955	1.3	1.160	2.3	0.045
0.4	4.650	1.4	0.905	2.4	0.030
0.5	4.285	1.5	0.700	2.5	0.020
0.6	3.875	1.6	0.525		
0.7	3.445	1.7	0.390		
0.8	3.010	1.8	0.280		
0.9	2.675	1.9	0.205		

If the density of the center strip be taken as unity (5.38 = 1.0), then the density of the strip 0.1 of the mean dispersion distant from the center of impact will be 5.33 ÷ 5.38 = 0.991; of the other strips 0.2 = 5.185 ÷ 5.38 = 0.964; 0.3 ÷ 4.955 ÷ 5.38 = 0.921, etc., as in the following table:

TABLE III. DENSITY FACTORS.

F	D	F	D	F	D
0.00	1.000	1.0	0.402	2.0	0.026
0.1	0.991	1.1	0.334	2.1	0.019
0.2	0.964	1.2	0.270	2.2	0.012
0.3	0.921	1.3	0.216	2.3	0.008
0.4	0.864	1.4	0.168	2.4	0.006
0.5	0.796	1.5	0.130	2.5	0.003
0.6	0.720	1.6	0.098		
0.7	0.640	1.7	0.072		
0.8	0.559	1.8	0.052		
0.9	0.479	1.9	0.038		

RULE: To determine the density of hits at any point in the group, divide the distance of the point from the center of impact by the width of the mean dispersion, both expressed in the same unit of measure as yards, inches, etc. This gives the "Density Factor" (F) opposite which in the table of density factors (Table III) will be found the density of hits at the given point in the group (D) expressed in terms of unity for the center strip.

EXAMPLE: Three wall targets are placed one behind the other, separated by distances of 25 yards. The center of impact lies in the center of the center target,

where 45 hits are expected out of each hundred, computed as previously explained. If the mean longitudinal dispersion (of the horizontal shot group) is assumed to be 50 yards, then

$$\frac{25 \text{ yards} = \text{the distance of target B from target A}}{50 \text{ yards} = \text{the mean longitudinal dispersion,}} = 0.5$$

and, from the table of densities, 0.5 = 0.796; that is to say, for every 100 hits expected at the center of impact (target A), 79.6 hits would be found on target B. Since 45 hits are expected on target A, by the statement of the problem, $45 \times 0.796 = 35.8$ hits would be found on either of the targets B or C (Fig. 29). The results thus obtained are only approximately true,

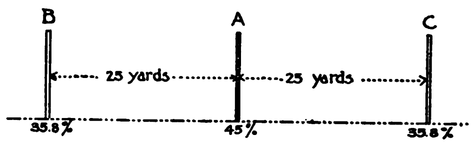


FIG 29.

as they are based on the trajectory being a straight line, but they are close enough for the majority of examples.

Graphically the density (or probability) curve (Fig. 30) shows the section through the "Hill of shots" which would be formed if every shot fired stopped in place where it struck. The smaller the group the steeper would be the "hillsides" and the less the prospect of hitting a given target if the center of impact does not coincide with the center of the target, conversely, the larger the group the gentler the hillsides and the greater the chance of hitting the target, but, of course, with a diminished number of projectiles.

That the computed curve is correct practically, can be demonstrated by use of the *Galton Quicunx* (Fig. 31.) This is a box with glass front with a number of pins (A) arranged in quicunx formation as shown, a funnel arrangement at the top (B) and a series of stalls (C) at the bottom. A charge of mustard seed (chosen because of its lightness and spheroidity) is poured into the top. The spheroids strike against the pins so as to scatter them in what appears an arbitrary manner.

The seeds fall into the stalls at the bottom in a

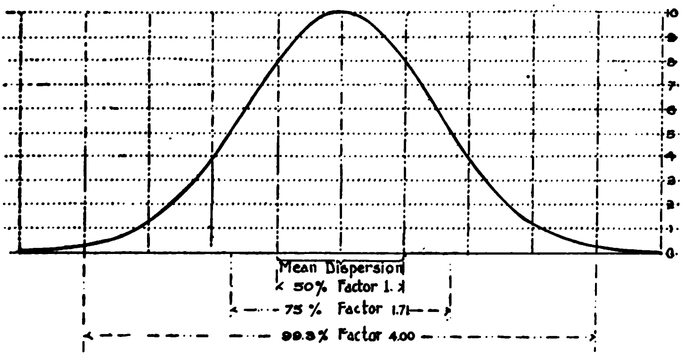


FIG. 30.

very close approximation to the computed curve which has previously been drawn on the glass face of the box.

DISTRIBUTION OF FIRE.

We have seen that the first step in estimating fire effect is a consideration of the probable per cent of hits on a target wall, and that the second step goes beyond this and estimates the number of hits to be expected on the figures of the battle target. Such a measure of comparison of fire effect serves very well for the technical

discussion of fire, but we cannot measure *tactical* results in this way.

A large number of hits may be so concentrated on the target that, while the percentage of hits is large, the *number* of figures hit is small, and many of the figures will not be hit at all, consequently the battle efficiency of the fire will be small. *For it is not the per cent of hits,*

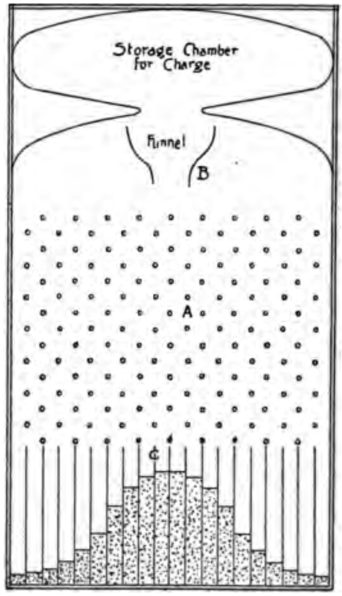


FIG. 31.

*but the number of men disabled, that measures results in war.**

* A familiar example of ignoring this principle is the collective fire practice of our present firing regulations, where the object sought is not to disable a large number of figures—as it would be in war—but to attain a large number of hits. The two are quite dissimilar as will be shown and the measure of comparison now in use (percent of hits) naturally encourages false instruction in the matter of group firing.

Now, if the fire is equally distributed over the whole target, the greatest possible number of figures will be hit (the *percentage of hits* to shots fired will, of course, remain the same), but even if the fire is equally distributed many targets will be hit more than once, and some will not be hit at all unless the number of hits is very small in proportion to the number of figures. That is, in an equally distributed fire, the number of disabled figures depends upon the ratio of the number of hits to the number of figures contained in the target.

For example, let us suppose that the target is composed of 100 figures, and at each volley or series of shots 5 per cent of the figures are hit. After the first volley there will be 5 figures hit and 95 figures that have not been hit. With the second volley, 5 figures are again hit, but, as the number of unhit figures was 95, it is evident that the possibility of hitting 5 new figures is diminished. Mathematically, only 4.75 unhit figures would be struck on the second volley, while 0.25 figures would be hit a second time. There remain, therefore, 90.25 unhit figures.

At the third volley 4.5 new figures would be hit for the first time, and 85.75 figures will not have been hit at all. Without entering into the demonstration farther, it is sufficient to say that a table may be constructed which will show how many hits (N) must, on the average, be obtained in each figure in order that a certain percentage (Z) of all the figures in the target may be hit at least once; and how many per cent of figures hit (Z) may be expected when every one is hit on the average N times.

The table presumes that the probability of being hit is the same for all figures, that is, that the fire is equally distributed over the whole object, and that all of the figures present equal vulnerable areas. These conditions

TABLE IV.

RELATION OF HITS TO FIGURES HIT IN A TARGET.

N	Z	N	Z	N	Z	N	Z	N	Z
0.01	1.0	0.41	33.6	0.81	55.5	1.42	75.8	2.55	92.2
0.02	2.0	0.42	34.3	0.82	56.0	1.44	76.3	2.60	92.6
0.03	3.0	0.43	34.9	0.83	56.4	1.46	76.8		
0.04	3.9	0.44	35.6	0.84	56 8	1.48	77.2	2.65	92.9
0.05	4.9	0.45	36.2	0.85	57.3	1.50	77.7	2.70	93.3
0.06	5.8	0.46	36.9	0.86	57.7	1.52	78.1		
0.07	6.8	0.47	37.5	0.87	58.1	1.54	78.6	2.75	93.6
0.08	7.7	0.48	38.1	0.88	58.5	1.56	79.0	2.80	93.9
0.09	8.6	0.49	38.7	0.89	58.9	1.58	79.4		
0.10	9.5	0.50	39.4	0.90	59.3	1.60	79.8	2.85	94.2
								2.90	94.5
0.11	10.6	0.51	40.0	0.91	59.8	1.62	80.2		
0.12	11.3	0.52	40.5	0.92	60.2	1.64	80.6	2.95	94.8
0.13	12.2	0.53	41.1	0.93	60.6	1.66	81.0	3.00	95.0
0.14	13.1	0.54	41.7	0.94	60.9	1.68	81.4		
0.15	13.9	0.55	42.3	0.95	61.3	1.70	81.7	3.25	96.4
0.16	14.8	0.56	42 9	0.96	61.7	1.72	82.1	3.50	97.8
0.17	15.6	0.57	43.4	0.97	62.1	1.74	82.4		
0.18	16.5	0.58	44.0	0.88	62.5	1.76	82.8	3.75	98.8
0.19	17.3	0.59	44.6	0.99	62.8	1.78	83.1	4.00	98.7
0.20	18.1	0.60	45.1	1.00	63.2	1.80	83.5		
								4.50	99.0
0.21	18.9	0.61	45.7	1.02	63.9	1.82	83.8	5.00	99.3
0.22	19.7	0.62	46.2	1.04	64.7	1.84	84.1		
0.23	20.5	0.63	46.7	1.06	65.4	1.86	84.4	5.50	99.5
0.24	21.3	0.64	47.2	1.08	66.0	1.88	84.7	6.00	99.8
0.25	22.1	0.65	47.8	1.10	66.7	1.90	85.0		
0.26	22.9	0.66	48.3	1.12	67.4	1.92	85.3		
0.27	23.7	0.67	48.9	1.14	68.0	1.94	85.6		
0.28	24.4	0.68	49.4	1.16	68.6	1.96	85.9		
0.29	25.2	0.69	49.8	1.18	69.3	1.98	86.2		
0.30	26.0	0.70	50.3	1.20	69.9	2.00	86.5		
0.31	26.7	0.71	50.8	1.22	70 5	2.05	87.1		
0.32	27.4	0.72	51.3	1.24	71.1	2.10	87.7		
0.33	28 1	0.73	51 8	1.26	71.6	2.15	88.4		
0.34	28.8	0.74	52.3	1.28	72.2	2.20	88.9		
0.35	29.5	0 75	52.8	1.30	72.7	2.25	89.5		
0.36	30.2	0.76	53 2	1.32	73.3	2.30	90.0		
0.37	30.9	0.77	53.7	1.34	73.8	2.35	90.4		
0.38	31.6	0.78	54.2	1.36	74.3	2.40	90.9		
0.39	32.3	0.79	54.6	1.38	74.8	2.45	91.4		
0.40	32.9	0.80	55.0	1.40	75.3	2.50	91.8		

If every figure in the target is hit N times on the average, then from the table, Z per cent of the figures will be hit.

will never, of course, be met in the service firing of infantry, for infantry fire will never be as equally distributed as the theoretical "equal distribution," nor even as evenly distributed as artillery fire, nor will all of the targets be equally exposed, because of folds in the ground, and other features which will make one part of the target less vulnerable than another. The effect of this will be to reduce the expected number of figures hit as determined from the table, though artillery often approaches it very closely.*

With an equally distributed fire delivered against a target composed of 75 figures and with such an accuracy that 90 hits are expected on the target, then every figure will be hit on an average $90 \div 75 = 1.20$ times (N), and with this average number of hits, we see from the table that 69.9 per cent of all the figures will be hit (Z). Since there are 75 figures in the target, 52.4 or 53 figures should be hit ($75 \times 0.699 = 52.4$).

Again, to disable 30 per cent of all the figures, 0.357 hits per figure must be made (from table), and since there are 75 figures in the target ($0.357 \times 75 = 26.78$), it is apparent that 26.78 hits must be made on the figures. Now, if it has been computed that 15 per cent

* In a series of artillery firings made at Okehampton by the English in 1893, a field battery fired 56 shells at 1,000 yards at a target composed of 90 kneeling figures, 200 yards behind which were 70 standing figures, and 300 yards in rear of that again a line of standing figures in close order with a frontage of 70 files. The object of the firing was to determine certain matters respecting the vulnerability of supports, but that the fire was distributed very equally over the whole target is evidenced by the results in figures hit compared with the "expected" number of figures hit as computed mathematically.

First line.	327 hits on 84 figures	(computed =87.5 figures)
Second line.	82 hits on 44 figures	(computed =48.3 figures)
Third line.	22 hits on 13 figures	(computed =19.6 figures)

of hits may be expected on the figures of a certain target by average marksmen at a given distance, and it is desired to disable 30 per cent of the 75 figures comprising the target, we can foretell the number of rounds which each man must fire, for it is evident that if we expect 15 hits from every 100 rounds and need only 0.357 hits on the average, we will have to fire enough rounds at a rate of 15 hits per one hundred shots to effect 26.78 hits ($75 \times 0.357 = 26.78$), or $26.78 \div 15 = 1.78 \times 100 = 178$ shots; so that if we have 25 men in the detachment, each man must fire $178 \div 25 = 7.1$ shots. With 50 men, each would fire 3.5 shots and with a number of firers equal to the number of figures (75) each would fire 2.38 shots.

Since this is a frequent and basic assumption, it is well to note that the computation becomes simply $(0.357 \times 100) \div 15 = 2.38$ when the number of firers is equal to the number of figures, and that this equation varies in the inverse ratio of men to figures, e. g. with one-third the number of firers each will require 3 times as many rounds $(0.357 \times 100 \times 3) \div 15 = 7.1$, and with three firers to each target, only one-third the basic number of rounds need be fired, or $(0.357 \times 100) \div (15 \times 3) = 0.793$ rounds per man.

ESTIMATION OF DISTANCE.

The effect of an error in estimating the elevation to be used against a given target is to raise or to lower the center of impact with reference to the point of aim on a vertical target; or on a horizontal target, to bring the center of impact closer to or farther from the firers than is the target itself. The amount of this movement in the case of the vertical target can be determined from

the table of ordinates and in the case of the horizontal target it is given in the statement of the problem. For example, let us assume that a vertical target stands at 1,000 yards from the firers and that the atmospheric conditions are such as to make the correct elevation exactly 1,000 yards. The center of impact will coincide (vertically) with the point of aim when an elevation of 1,000 yards is used. But if an elevation of 1,100 yards is used, then the mean trajectory will be 8.317 feet above the line of sight at the target since the 1,000 yard ordinate of the 1,100 yards trajectory is 8,317 feet. The center of impact would therefore be raised 8.317 feet above the point of aim in this case. If an elevation of 900 yards was used the center of impact would lie 11.949 feet *below* the point of aim on the 1,000 yards target for the 1,000 yards ordinate of the 900 yard trajectory is —11.949 feet by the table of ordinates. In the table the minus ordinates are given for distances up to 150 yards beyond the target. Where greater distances are involved the ordinates may be determined approximately by interpolation, or, when fractional parts of the tabular distances are required they may be interpolated. Interpolation, unless the distances involved are small, introduces minor errors but is sufficiently close for the purpose.

With the displacement of the center of impact known the computation of expected hits is made as previously explained. With the error of the range stated in yards, a close approximation may be made by the method of densities, using either Table III or Table V which, based on Table III has been elaborated by General von Rohne in his *Schliesslehre für Infanterie* and which makes the computation quite accurate enough for purposes of comparison or of discussion.

TABLE V. DENSITY RATIOS.

Considering the density of the hits in the center of impact as 100, the density at a distance of 25–50–75–yards etc., is — for a longitudinal dispersion of 30–35–40 yards etc.

Error in Setting the Sight —Yards—	Size of (50%) mean longitudinal dispersion in yards.																		
	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120
25	53	63	70	75	80	83	85	87	89	90	91	92	93	94	94	95	96	96	96
50	8	15	26	32	40	47	53	58	63	67	70	73	75	77	80	81	83	84	85
75	0.4	1.6	4	8	13	19	24	30	35	40	45	49	53	57	70	63	66	68	70
100	.	.	0.4	1	3	5	8	12	15	20	26	28	32	36	40	44	47	50	53
125	.	.	.	.	0.4	1	2	4	6	8	11	14	16	21	24	28	31	34	37
150	.	.	.	.	.	.	0.4	0.8	1.6	2.6	4	6	8	10	13	16	19	22	24
175	.	.	.	.	.	.	.	.	0.4	0.7	1.2	2.2	3.3	4.6	6	8	10	12	15
200	.	.	.	.	.	.	.	.	.	.	0.4	0.7	1.1	1.9	3.	4	5	6	8
225	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1.6	2.3	2.2	4
250	.	.	.	.	.	.	.	.	.	.	.	.	.	.	0.4	0.4	0.8	1.2	1.9
275	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	0.4	0.6	0.8
300	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	0.4

Error in Setting the Sight —Yards—	Size of (50%) mean longitudinal dispersion in yards.																		
	125	130	135	140	145	150	160	170	180	190	200	220	240	260	280	300	350	400	450
25	96	97	97	97	97	97	98	98	98	98	99	99	99	99	99	99	99	99	100
50	86	87	88	89	89	90	91	92	93	94	94	95	96	97	97	97	98	99	99
75	72	74	75	77	78	80	82	84	85	86	87	89	91	92	93	94	96	97	97
100	56	58	61	63	65	67	70	73	75	78	80	83	85	87	89	90	92	94	95
125	40	43	46	49	51	53	57	62	64	67	70	74	78	81	83	85	89	91	93
150	27	30	33	35	38	40	45	49	53	57	60	66	70	74	77	80	84	87	90
175	17	19	22	24	26	29	33	38	42	46	50	56	62	66	70	73	80	84	87
200	10	12	14	15	18	20	26	28	32	36	40	47	53	58	63	67	74	80	84
225	5.2	6	8	10	11	13	17	21	24	28	32	39	45	51	55	60	69	75	80
250	2.6	4	5	6	7	8	11	14	16	21	24	31	37	43	47	53	63	70	75
275	1.2	1.8	2.3	2.6	3.8	4.8	6.8	9	12	15	18	24	31	36	40	47	57	55	71
300	0.6	0.8	1.1	1.6	2.1	2.6	4	6	8	10	13	19	24	30	35	40	51	66	66

EXAMPLE: Against a target 1 yard high, good marksmen at 1,100 yards (correct elevation) expect 40 per cent. of hits. If they use an elevation of 1,050 yards (50 yds. error) the expected number of hits will be reduced by 0.36. The mean longitudinal dispersion is 47.75 yards—by interpolation—from Table II. With a dispersion of 45 yards—by Table V, and an error of 50 yards the density of the target would be .32; with a dispersion of 50 yards the density is 40; hence with a dispersion of 47.75 the density would be 0.36 of that at the correct elevation. 40 x 0.36 = 1.44 per cent.

If they use an elevation of 1,000 yards (100 yards in error), the expected number of hits will be reduced by 0.03 (mean longitudinal dispersion = 50 yards) therefore 40 x 0.03 = 1.20 per cent.

The results obtained by the use of the table are quite accurate enough for the usual computations and while they differ a little from results otherwise obtained, the difference is negligible.

DANGER SPACE.

The danger space is the distance measured along the line of sight within which the trajectory does not rise above the highest point of the target nor fall below its lowest point.

It is evident that the extent of the danger space depends upon the relation between the trajectory and the line of sight—the angle of fall—and therefore on the range and the curvature of the trajectory, on the height of the target and finally on the point of aim; that is, the point where the line of sight meets the target. (Fig. 32.)

If we consider a target with a height (h) placed in

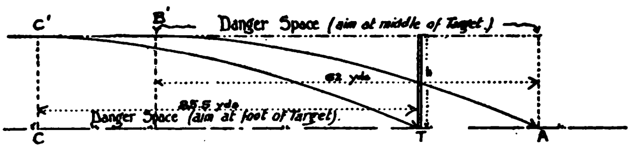


FIG. 32.

the line of fire, then the danger space beyond the target extends through the point of impact (middle of the target) to A (Fig. 32) where the trajectory has fallen one-half the height of the target below the line of sight. In front of the target it extends to a point (B′) where the trajectory has risen one-half the height of the target above the line of sight. The continuous danger space therefore extends from the point B′ (high strike) to A (ground strike), and it is evident that the location of the high strike and the ground strike determine the extent of the danger space.

By using an elevation of 700 yards against the middle point of a cavalryman, eight feet high, the danger

space extends from the firers to 801.5 yards. Against an infantryman, on the other hand, it extends from the 586.8 yard point (high strike) to 772.6 yards (ground strike) or a total distance of 185.8 yards. If the point of aim is taken at the *bottom* of the target in the foregoing cases, it is 700 yards long in each case; a noticeable gain in the latter case but a loss in the case of the cavalryman.

For the purpose of studying the effect of various aiming points, especially at the longer ranges, and assuming that the trajectory is straight for 100 yards, the danger space may roughly be computed in the following typical cases:

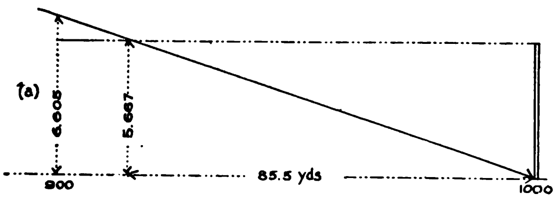


FIG. 33.

(a) The point of aim is at the foot of a standing man (68 inches).

(b) The point of aim is at the top of a standing man.

(c) The point of aim is at the center of a standing man.

(a) The 900 yard ordinate of the 1,000 yard trajectory is 6.605 feet. The target is 5.667 feet high. b = danger space — aim at foot of target.

$b : 100 : : 5.667 : 6.605.$

$b = 100 \times 5.667 \div 6.605 = 85.5$ yards. (Fig. 33.)

(b) b=danger space, aim at upper edge of target.

The 1,100 yard ordinate of the 1,000 yard trajectory = —9.149;

b : 100 : : 5.667 : 9.149.

b = 100 × 5.667 ÷ 9.149 = 62. (Fig. 34.)

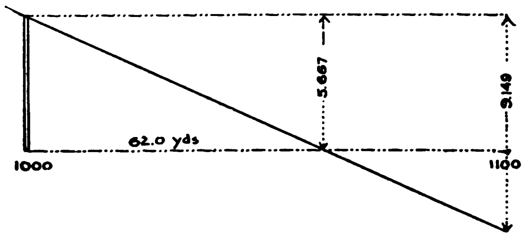


FIG. 34.

(c) b = danger space — aim at *center* of target.

b = mean of (a) and (b).

= $\frac{1}{2}$ (85.5 + 62.0) = 73.75, of which one-half lies in front and one-half in rear (36.88 yards each). (Fig. 35.)

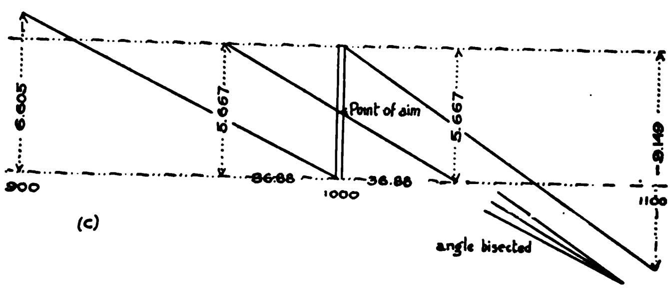


FIG. 35.

The manual gives 39.6 yards in front and 35.2 yards in rear, a total of 74.8 yards instead of 73.74 as above, but, of course, considers the curvature of the trajectory.

The danger space is greatest (85.5) with the point of aim at the bottom of the target, and smallest (62.0) with the point of aim at the top. It would be premature, however, to expect to gain any great advantage from the low point of aim. The principal reason for the difference lies in the fact that the longer the range the shorter the danger space owing to increasing curvature of the trajectory, and this agrees with the three cases just worked out where—

(*b*) The range extended from 1,062 to 1,000 yards (62.0).

(*c*) The range extended from 1,036.9 to 962.1 yards (73.75).

(*a*) The range extended from 1,000 to 914.5 yards (85.5).

The displacing of the center of impact from the center of the target is a factor which must also be considered and it will often be the controlling factor. The danger space at ranges under 700 yards is affected by the position of the firer (height of muzzle from the ground), the danger space increasing as the height of the muzzle decreases. At the longer ranges no material effect is felt from different positions of the firer.

In dealing with danger spaces, it is usual to distinguish between the *danger space* and the *swept space*, both of which are functions of the mean trajectory and between these and the dangerous zone which is a function of the whole, or a part of the cone of fire.

It will be observed that for a given height of target and point of aim, the danger space is of fixed dimensions at the several ranges, while the swept space varies in addition with the slope of the ground, being shorter on rising ground and longer on falling ground than the

danger space, and all the functions of the dangerous zone such as the density of the group at a given distance from the center of impact, etc., are correspondingly modified.* (Fig. 36.)

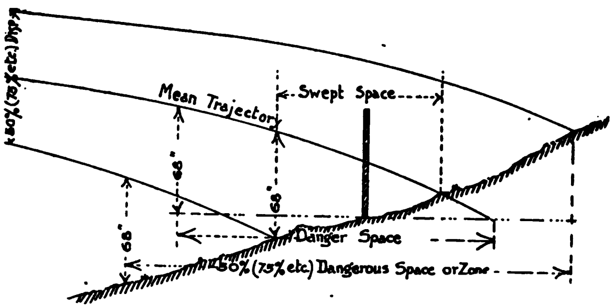


FIG. 36.

INFLUENCE OF THE GROUND.

In calculating the danger space, at any probable infantry range, the trajectory may be regarded as a straight line for a distance of 100 yards as stated, and the same presumption is warranted in computing the swept space. Since all parts of the trajectory are really curved, this introduces a small error and tends to give

*Von Rohne calls the danger space the "sight reach" (visierbereich) to distinguish it from the "swept space" (bestrichener Raum). The Austrian ballisticians distinguish the two by calling the danger space the "swept space" (bestrichener Raum) and the swept space the "modified swept space" (modifizieter bestrichener Raum). The French, with their more flexible language, make the distinction much as we have given it above, the danger space being called the "dangerous zone" (Zone dangereuse), and the swept space the "swept zone" (Zone rasee).

Neither the German Firing Regulations nor our own recognize any difference between the two spaces, but call both by one name (the "swept space" in Germany, and "danger space" in the United States).

spaces in excess of those obtained when the curvature of the trajectory is included in the computation. Dealing as we are here with the effect of fire rather than with the theory of ballistics, the error introduced by assuming that within the danger space the trajectory is a straight line is negligible.

In Figure 37 let—

A — B = Danger space = Swept space on ground parallel to line of sight = (d).

A — C = Swept space on rising ground = (s).

f = Angle of fall of trajectory, and

g = Angle of slope of the ground. f and g are measured in a vertical plane

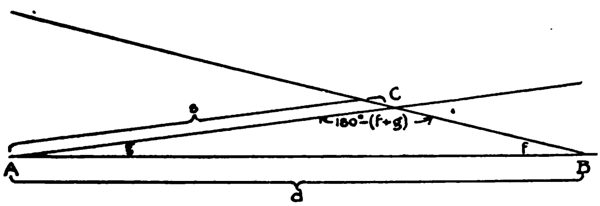


FIG. 37.

through the line of sight (*"Plane of fire"*) and are the angles which the last element of the trajectory and the slope of the ground at the target respectively, make with the line of sight.

Then, from the figure and since the sides of a triangle are proportional to the sines of the opposite angles,

(A—C or), $s : d :: \sin f : \sin [180^\circ - (f+g)]$,

and, since the sine of an angle is the same as the sine of its supplement,

$$s : d :: \sin f : \sin (f+g)$$

$$\therefore s = d \frac{\sin f}{\sin (f+g)},$$

and since the angles f and g are always small and the sines of small angles vary as the angles,

$$s = d \frac{f}{f+g}$$

Example: At 1,140 yards the angle of fall is 2°. If the ground at the target slopes upward 1° with reference to the line of sight, then the swept space becomes

$$s = d \frac{2}{2+1}$$
$$= d \frac{2}{3}$$

At 1,140 yards the danger space against an infantryman is about 56 yards, the swept space therefore is

$$s = 56 \cdot \left(\frac{2}{3}\right)$$
$$= 37 \text{ yards.}$$

Similarly on falling ground. (Fig. 38), let

A — B = Danger space = Swept space on ground parallel to line of sight = (d).
A — D = Swept space on falling ground = (s).
f = Angle of fall of trajectory.
g = Angle of slope of ground.

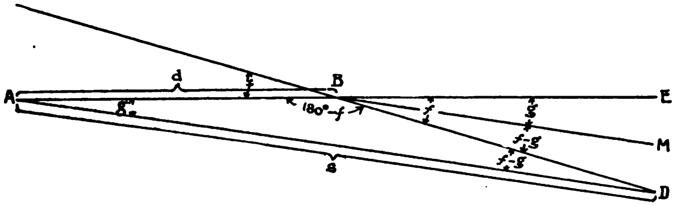


FIG. 38.

In Fig. 38, it will be observed that in the triangle ABD, the angle at B is 180° — f. That the angle BDA is equal to (f — g). For BM is parallel to AD = slope of ground. Hence the angle EBM = EAD = g. And since the angle EBD = f, MBD = f — g = BDA.

Therefore, since the sides of the triangle ABD are proportional to the sines of the opposite angles,

AD : AB : : sin ABD : sin BDA.

or s : d : : sin (180° — f) : sin (f — g),

or s : d : : sin f : sin (f — g), and since the angles are small,

$$s : d : : f : (f - g),$$

$$s = d\frac{f}{f-g}$$

At 1,140 yards, d has a value of 56 yards, as stated, and if the ground slopes downward 1° with reference to the line of sight, so that g = 1°, then

$$s = 56 \times \frac{1}{2-1}$$

$$= 56 \times 2 = 112 \text{ yards.}$$

If the ground slopes 2°, the trajectory which has the same angle of fall would be parallel to the ground and would eventually strike the ground only because it

is really a curved line instead of the straight line which we have assumed.

If the ground slopes downward at a greater angle than 2°, then a low object at a short distance beyond 1,140 yards would not be hit at all. (Fig. 39).

FIG. 39.

In general, on falling ground, when the value of g = 0, the swept space is the same as the danger space; if g is less than f, the swept space exceeds the danger space; if g = f, the swept space is (theoretically) infinite; if g is greater than f, a dead space is formed as in Fig. 39.

At the short ranges f is less than at the long ranges, hence the influence of a given slope is greater at the short ranges than at the long ranges. For the purpose of comparison, assume a rising slope of 2°. At 834 yards the angle of fall is 1°. The swept space is $d \times 1/1 + 2 = 1/3 = 3/9d$.

At 2,060 yards the angle of fall is 7°. The swept space is $d \times 7/7 + 2 = 7/9d$.

Irrespective then of the actual shortening in yards, it will be seen that the 2° upward slope which at 834 yards reduced the danger space 6/9, at 2,060 yards only reduced it 2/9, or that at the shorter range the influence of the ground was three times as potent as at the longer range.

Considering falling ground (2°).

At 834 yards the angle of slope (g = 2°) is greater than the angle of fall (f = 1°), hence a dead space exists and the swept space is 0.

At 1,140 yards the angle of fall is 2°, hence g=f and theoretically an infinite swept space exists.

At 2,060 yards the angle of fall is 7° and the swept space is 1.4 greater than the danger space. (7/7 — 2 = 7/5 = 1.4.)

Here again the influence of a given slope is greater at the short ranges than at the long ranges. These figures show that the influence of the ground is more important as the trajectory is more rasant and that the modern flat trajectory has greatly increased the influence of sloping ground.

As applied to the density of the shot group, the formulæ are exactly the same as those just given. For example, with a 1° angle of fall and a 1° upward slope of the ground, the mean longitudinal dispersion (or any other function of the shot group) becomes—

$$
\begin{aligned}
D_m &= \frac{f}{f+g} \\
&= \frac{1}{1+1} \\
&= \frac{1}{2}
\end{aligned}
$$

So that with a tabular mean longitudinal dispersion of average marksmen (at 834 yards) of 92 yards, the mean longitudinal dispersion on a 1° rising slope is $\frac{1}{2} \times 92 = 46$ yards.

On ground falling 1°, g = f and the mean longitudinal dispersion becomes infinitely great.

On level ground, at 750 yards, the mean longitudinal dispersion of average marksmen is 100 yards. If a line of skirmishers be placed at this range and lines of supports be placed in rear at distances of 100 yards, it is evident that with the center of impact at the first line,

the density of the shot group at that point would be 1.0; 100 yards in rear it would be 0.402; 200 yards in rear 0.026, etc. (Table III.) (Fig. 40.)

If the ground slopes upward at the same degree as the angle of fall, then Dm $= 1/1 + 1 = \frac{1}{2}$, and the dens-

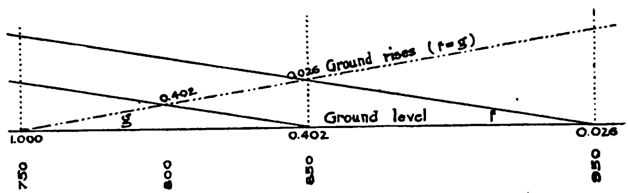


FIG. 40.

ity 100 yards in rear is only 0.026, while 200 yards in rear no hits at all are expected.

A somewhat similar problem is that involved in the determination of the effect of a sloping terrain upon which the target stands.

Against a wall target, a single line of skirmishers, etc.,—a target without depth—the slope of the ground upon which the target stands does not affect the number of hits to be expected upon the target, but with troops in column or other deep formations, the slope of the ground upon which the target stands does become important and the deeper the target the more is this influence felt.

If we assume that a target wall stands at T (Fig. 41), it will be made apparent by the figure that it makes no difference whether the ground upon which it stands is level or is inclined upward (A — B) or downward (C — D), provided that the center of impact is at T. However, if the target standing on ground parallel to the line of sight has depth, as where it extends rear-

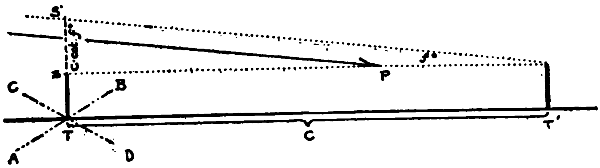


FIG. 41.

wards a distance C or from T to T′, then some of the bullets which pass over the top of the target at T will strike in the top of the target between T and T′ as at P. The effect of this is simply to increase the vertical height of the target at T by C times the tangent of the angle of fall, or, graphically, from S to S′.

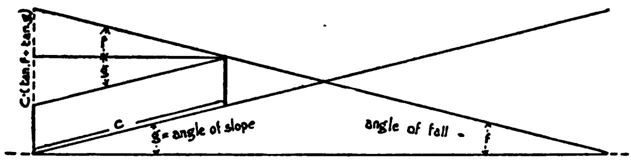


FIG. 42.

If, instead of being on level ground a deep target such as a column of troops stands on ground sloping upward with reference to the line of sight, then, from Fig. 42, the vertical target is increased by its depth multiplied by (tan f + tan g).

Example: A company in column of squads is standing on a rising slope of 3° (depth of column 60 yards), and at 2,060 yards from the firers (angle of fall 7°). The height of the target, then, is not 68 inches but 68″ + 60 yards × (tan 3° + tan 7°) = 68″ + 2160 inches (0.0525 + 0.124) = 68 inches + 382 inches = 450 inches.

On falling ground, as where the troops in column are held in reserve on the reverse slope of a hill, the vertical target presented is lessened by the downward slope until the slope and angle of fall are equal at which point only the men in the front rank could be hit (disregarding penetration and curvature of the trajectory)

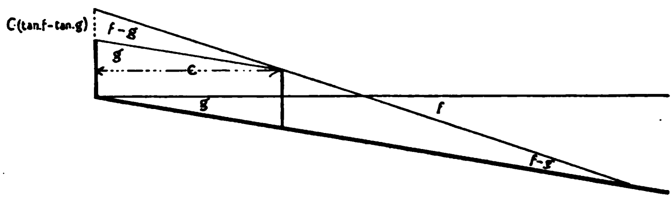


FIG. 43.

as all high misses would also miss the men in rear of the front rank, and the vertical target would be 68 inches high. On all such slopes, that is where the angle of fall exceeds the angle of slope, the vertical target presented becomes the height of the target plus the depth multiplied by the difference of the tangents, or T + C (tan f — tan g).

Using the same slope and angle of fall, but on a downward slope (— 3°), the increase in vulnerability would be 60 yards (tan f — tan g) = 2160 inches (0.124 — 0.053) = 155 inches, and the whole column would have a vulnerability equal to a vertical target 223 inches high. (Fig. 43.)

CHAPTER III.

Modern fire effect should be studied from two points of view, the psychological and the physical, and since the former is more or less involved in the latter, it is well in passing to consider the theory upon which the psychological effect rests.

Fire effect manifests itself psychologically by producing either a stunning and paralyzing effect or a gradual exhaustion, disintegration and wasting away of the hostile strength. Fire effect is stunning or paralyzing when it is very much concentrated both as to time and space, for through this concentration and suddenness it produces fear and terror. The effect of fear is to arrest the fighting impulse, through the opposing impulse to escape an appreciated danger; while terror, automatically or reflexively invokes a paralyzing impulse without a clear consciousness of what the danger is.

The effect, then, of a highly concentrated sudden burst of rapid fire which produces both fear and terror is often utterly to destroy the fighting capacity of the troops subjected to it. On the other hand, a gradual disintegration, wasting away or destruction of the hostile force takes place when the fire action is more extended both as to time and space. When the fire fight takes this form, the material losses suffered and the gradual exhaustion of the physical energies induce the conviction among the troops subjected to it, that they cannot hope for a successful result, and this conviction causes a suspension of the power to act and with it, of course, the fighting will. Psychologically it is as though

one were bound hand and foot so that he forms a conviction that he cannot walk. The forming of this conviction includes also the negation of any intention to walk, so that he will not *intend* to walk out of the room. Similarly, when the conviction seizes the defending troops that they cannot stop the advance of the attacking enemy, the *intention* to stop him also ceases and the fighting will is destroyed.

In the modern fire-fight both of these effects are sought by merging one into the other. A concentrated and sudden burst of fire which effects no physical results—a mere whistling and cracking of bullets over the heads of the troops—will not as a rule produce the stunning effect desired, but if the gradual disintegration of the hostile forces is combined with the paralyzing effect of the fire storm, the result is materially increased. In a general way the progress of the fire fight in the attack might be said to consist of a gradual breaking down of the fighting will of the defenders by a deliberate and uniformly distributed fire, and then the bringing about of the decision by a psychically stunning rapid fire which, while producing little physical effect against the small targets presented, utterly paralyzes the fighting will and makes the mere threat of cold steel sufficient to force the defenders to evacuate their position.*

While the psychological effect of fire is undoubtedly a great factor in the result and one which should be studied and understood by all officers, the scope of this

* The psychology of fire is covered exhaustively in Creuzinger's "Problems of War, Part I," and in Maude's "Evolution of Infantry Tactics," and is both discussed and historically illustrated in Minarelli's "The Actions in Natal and The Cape Colony—1899," as well as by older and perhaps better known writers such as von der Golz and Clausewitz.

work will hardly permit an inquiry into a subject which is so large that many fair sized volumes have been devoted exclusively to it, and which because of its importance and extent is made a special study in the war colleges of all the Continental countries. Here we will content ourselves with an inquiry into the physical effects of fire and into the methods by which that fire superiority may be attained, which is an indispensable condition of success, both in offensive and in defensive operations, and which in the final analysis is the deciding factor of all war.

Fire superiority can be attained only by inflicting upon the enemy in a given space of time more losses than he himself occasions. All instruction in firing should have for its ultimate aim the fitting of the army to attain superiority of fire in battle, and any fire instruction which does not contribute to this end to a commensurate degree is wasted. Since excellence in marksmanship is to be judged upon the basis of its effect under service conditions, it may happen that a technically excellent fire may produce such low results measured by service standards that tactically it is useless; hence the comparison from a military point of view of the results attained by men or organizations where target practice is conducted along the lines of the civilian shooting club is as false and misleading as are the expectations as to its war value which such shooting too often raises in the minds of the uninitiated.

The effect of rifle fire in war is measured by the number of men disabled in a given space of time. It is dependent upon:

(1) The percentage of hits, which depends on:

(a) The dispersion, which is variable with the weapon, distance, visibility of target, atmospheric conditions, and especially with the marksmanship of the firers (variable with the training, physical condition and the morale of the men).

(b) Upon the designation of the objective, and estimate of distance.

(c) Upon the character of the objective (number of figures, nature, disposition, extent of front, visibility, vulnerable area, etc.).

(d) Upon the character of the ground at the target—favorable for riccochets, observation of fall of bullets, and—for echelons in rear, the slope of the ground in rear of the target.

(2) The number of rifles engaged.

(3) The amount of ammunition expended.

(4) The rapidity and duration of the fire.

(5) The control of the fire, its distribution, the choice of objective and fire discipline.

(6) The effect of the hostile fire.

SIZE OF THE DISPERSION.

The size of the mean dispersion is the foundation upon which all study of fire effect is based, it is the width of a strip which, under the various conditions of arms, range and marksmanship, will contain 50 percent of all the shots fired, 25 percent lying on each side of a central line through the center of impact.

With modern rifles and ammunition in good condition, the dispersion due to the weapon alone will be very

small even at the longer ranges. It will be larger with the same rifle after it has been fired a great number of times and larger with an old type of rifle even when in good condition than with the modern rifle if still in a serviceable condition.

For purposes of comparison in the performance of different arms the 50 percent radius forms a convenient and usual measure. The 50 percent radius is the radius of a circle having its center at the center of impact and a radius (50_r) of such a dimension that the circle will contain half of all the shots fired.* It is sometimes used as a basis for the study of fire effect, but should not be confused with the "mean radius of the circle of shots," which is a measure of accuracy used by gunmakers.

Comparing the size of the 50 percent radius at 1,000 yards in the last three models with which our army has been supplied (no data has yet been obtained as to the dispersions or radius of the '03 rifle with the '06 bullet), it will be seen that the improvement in the rifle has materially decreased the dispersion which may be attributed to that source alone.

	Cal. 45	Mod. '99	Mod. '03
50_r for single rifle.	22.5	17.8	15.8
50_r for group of rifles. .	(31.8)	(18.9)	(17.9)
Comparative hits.	19.8	46.5	50.

But because of the greater dispersion in each arm due to differences in the correctness of the sighting parts, the dispersion in each case due to "Armament" is larger as shown in the bracketed figures. Against a target with

*From a known vertical and lateral dispersion, the valve of 50_r may be found from the formula $50_r = 0.88 \times \sqrt{d_v \times d_l}$; wherein d_v is the mean vertical dispersion and d_l the mean lateral dispersion. *Von Rohne.*

a bullseye about 36 inches in diameter (radius 17.9 inches), 50 out of each 100 shots would hit the bullseye with the '03 model fired from a vise; 46.5 hits from a model '96, and only 19.8 hits with a caliber .45. So far as the type of rifle and ammunition is concerned, then, the improvement in the past twenty years means that two and one-half times as many hits may now be made as with the older type of rifle with no increase in the skill or training of the marksman.

But the accuracy life of the present rifle falls off rapidly after several thousand rounds have been fired from the same rifle, and this causes an increase in the dispersion over which the marksman can have no influence. For example, the 50 percent radius of the '03 rifle is 15.85 inches at 1,000 yards with a new gun. After firing

2,000 rounds, the 50 percent radius becomes. . . . .(25.5) 23.8 inches;
4,500 rounds, the 50 percent radius becomes. . . . .(46.5) 45.7 inches, and 20 per cent of the bullets will reach the target;
5,000 rounds, the 50 percent radius becomes. . . . .(67.8) 67.2 inches,

in a favorable case, and in many cases the dispersion would be so large that the radius could not be determined.

The effect on the percent of probable hits of this deterioration of the rifle is shown by the fact that where with the new rifle 50 hits were expected, after 2,000 rounds there would be probably only 29.0 hits, after 4,500 only 10 hits, and after 5,000 rounds only 4.8 hits. And this without any decrease in the skill of the marksman; hence in war the fire effect of the troops may be

expected to fall off rapidly with the age of the rifle. Troops armed with old rifles (5,000 rounds) would have to fire ten times as many cartridges to attain a certain result as would be required with new guns, and would be under hostile fire for ten times as long as would otherwise be necessary in order to fire those rounds.

With a uniform class and condition of rifles and ammunition and marksmanship the dispersion increases with the range but not in a direct proportion as will be seen from Table II. Of the influences which cause the dispersion, those due to meteorological conditions, variation in initial velocity, resistance of the air, etc., increase in approximately a geometric ratio with the range while those due to a lack of skill in the firer are angular errors and increase directly as the range. The combined effect of these influences causes an increase in the dispersion only a little more rapid than the change in the range at short distances and much more rapid than the change in the range at the long distances. This increase in the dispersion is due to two quite distinct general causes: *First.* Variations in the arms and ammunition and meteorological changes. *Second.* Variations due to lack of skill in marksmanship. Of these two, the first is of very small importance at ranges under 500 yards as compared with the second, beyond that range, and in an increasing degree, the first becomes the dominating influence. Marksmanship is therefore considered under two heads—the mechanical and the intellectual.

The mechanical part of shooting consists of aiming correctly and uniformly, holding the point of aim at the same point on the target during successive shots and firing the piece so as to avoid disturbing the aim. These

may all be taught in a short time by preliminary instruction (aiming and position drills and gallery practice), followed by range practice at a suitable bullseye target placed at a short distance from the firer where practically all the dispersion is due to the firer and the cause of a misplaced shot can be explained to the soldier and corrected.

The intellectual part of shooting consists of so training the judgment of the meteorological conditions that the firer will be able to make the necessary changes in the setting of the sights, point of aim, etc. If, for example, firing is conducted on a very hot day, the soldier by reason of this intellectual training will know that his sights must be set at something less than the true range, and according to the degree of skill which he has attained, he will be able to judge the amount of the reduction which the atmospheric conditions demand. Having set his sights correctly and chosen the correct point of aim, the mechanical part of shooting is not different from that at the short ranges, except that it exercises less influence on the resulting dispersion. Since individual soldiers will not (except in abnormal situations) ever fire at the longer ranges and so will never have an opportunity to exercise judgment in the matter of the allowances to be made for atmospheric conditions, it follows that the mechanical part of the shooting instruction is all that is necessary for the soldier to master, and it is for this reason that individual instruction should not be given at ranges greater than 500 yards. As firing in battle under modern conditions—especially when on the defensive—is often conducted at ranges of 2,000 yards or even more, it follows that some instruction should be imparted at ranges greater than that limiting profitable individual

instruction. Firing at ranges over 400 yards or 500 yards will always be by group and under the direction and control of officers and non-commissioned officers, hence the instruction imparted at the longer ranges should have in view the training of the leaders rather than that of the individual soldiers. (When the soldier in the heat of battle gets so out of hand as to be unable to respond to the orders of his group commander and must perforce take up individual fire, it is evident that his dispersions will be so large that any thought of refinement in shooting is out of the question.)

The dispersions in Table II are based upon group firing results throughout and upon the existence of favorable target range conditions; in field firing where the individual cannot fire as calmly as when at target practice, where the position is frequently off-hand, where a fatiguing march has, perhaps, just preceded the firing, where the targets are poorly lighted and partially hidden, where hostile shrapnel is bursting overhead and his comrades are falling about him, all the individual errors in aiming will be increased. Such errors are angular errors and increase in magnitude more rapidly than the range; moreover, they vary in degree with the different soldiers composing the firing group, with the sense of danger, etc., and united, they increase the dimensions of the dispersions until all difference is lost between the dispersions of trained and untrained marksmen and the distinction becomes one between the undisturbed and the excited, all of whom are composing the firing group and uniting to produce a large dispersion.

This subject has been exhaustively investigated by the well known Russian ballistician Wolozkoi (*"Infantry Fire in Action"*), and while the final deductions of

that author cannot be accepted unreservedly, his experiments are useful in computing the effect of probable angular errors.

After very lengthy and carefully conducted tests extending over a considerable space of time and with thousands of men, Wolozkoi determined that good marksmen make an average angular error of ± 8 minutes, poor marksmen ± 40 minutes, and that the average of a mixed command is ± 25 minutes.*

If, however, we assume that these errors of the firers amount to a mean of only ± 2½ minutes (= 5 minutes of arc), the mean dispersion would be increased by reason of the erroneous aiming from 72.5 inches (Table II) to 89.4 inches (Didion's Law).† Against a target

* The Wolozkoi theory in brief is that when men are under severe fire they will produce what he calls a "Constant Cone of Misses," that is, that the sheaf or cone will have unvarying dimensions and will not be dependent upon the target or range. He fixes the lowest trajectory of the cone as having an elevation of 1 degree and 30 minutes with the horizontal and the highest trajectory as having an elevation of 6 degrees and 30 minutes. By applying these figures to the present rifle ('06 bullet) we would have an uninterrupted beaten zone extending from 1,400 yards to 2,800 yards, or a mean longitudinal dispersion of 350 yards. It should be borne in mind, however, that Wolozkoi is dealing only with men who are under fire themselves, are hurried and nervous, and who consequently aim badly. Whether correct in his deductions or not, it cannot be denied that his reasoning is in entire accord with the history of fire effect in battle. The principal objection which is raised to his theory is that when men are in the condition which he assumes, the results of their fire cannot be discussed mathematically. The psychological factor which cannot be reduced to figures is admittedly the controlling factor and it is generally held that Wolozkoi's theory remains a theory rather than a demonstration.

† A mean error of 5 minutes in aiming causes a vertical dispersion of 52 inches at 1,000 yards. The mean vertical dispersion of average marksmen at this range is 72.5 inches (Table II). The combined dispersion would be $\sqrt{72.5^2+52.0^2}=89.4$ inches.

wall 36 inches high, this would mean a decrease in hits per hundred of from 26.4 for average marksmen to 10.5 for the same marksmen when making the angular errors considered, *i e.* $\pm 2\frac{1}{2}$ minutes.‡

The dispersion in field firing will be greater, as has been said, than those given in the table, due to the disturbing elements which surround the firers. To a large degree the extent of this increased dispersion will be due to a lack of training in the men. It will be very great with recruits and much less in organizations composed of seasoned men and trained marksmen. The presence of the enemy will, in itself, increase the dispersion, but it should be observed that while the enemy is still distant and the danger less, the men will retain a certain calmness and a correspondingly small dispersion, still when the hostile lines approach, the increased sense of danger will cause the dispersion of any given class of soldiers to be greatly increased, for not only will the angular errors be greater than those made at the longer ranges, but the influence of an error of this kind is much greater at the short ranges than at the long. Thus the natural tendency to a decreased dispersion at the shorter ranges will be neutralized by a greater dispersion due to what the English aptly call "Nerves."

The importance of the visibility of the target is almost self-evident. If one is firing at a black bullseye on a white paper target, one can readily insure that the angular errors shall be small, for any difference in aiming or holding becomes at once apparent. If, however, one is firing at a skillfully hidden line of heads, or, as

‡ An angle of 2½ minutes can be appreciated when it is understood that such an angle is produced when, in aiming, 0.016 inch more or less of the front sight is seen. (0.016 inch is a little less than one-third of the thickness of the front sight leaf.)

often occurs in battle, at no visible target, the ability to aim correctly and to hold on the same spot in successive shots is rapidly lost and very large angular errors are introduced even when the men are as calm and collected as when on the target range. The degree of visibility of the target, then, is one of the most important factors in determining the size of the dispersion and it manifests itself in three different ways:

(1) The power to see the target and the existence on the target of a suitable and clearly defined aiming point.

(2) The power of judging distances.

(3) The power quickly and accurately to recognize a target when indicated by an officer or non-commissioned officer.

In all of these, the acuteness of vision of the soldier plays an important part and the training of a soldier's vision occupies a very prominent place in his instruction in firing in all of the leading armies of the world. City bred recruits have, as a rule, very poor vision at anything but short distances, but by training—eye gymnastics—their vision can be greatly improved and the limit of vision easily extended to extreme infantry ranges. If, when firing at the invisible targets which the battle ground of today presents, one cannot see the enemy, it is apparent that one cannot aim at him. If, however, the vision has been trained to a keenness which permits the detection of a skirmish line even when partly concealed, a target exists and a greater result may be expected from the fire. The dispersion of such men will necessarily be smaller than that of men who are firing at an unseen target. In the usual case we will have to deal with men of only very limited powers of vision

and the enemy will be absolutely invisible to the majority of the command, who will, therefore, be forced to aim at some self-chosen spot on the landscape.

The "Void of the Battlefield" will be one of the principal causes of the great dispersions to be anticipated in war and it is evident that anything which will decrease the influence of such a cause is of the utmost importance. It was a common thing among the English in South Africa with new troops to hear the men say after a battle, "I never saw anyone, I never fired a single round," but at the close of the war these same men had acquired almost as acute a vision as the Boers themselves, and generally were able to see the battle targets. During the progress of a war is a very poor time to impart instruction in this important part of training in musketry, for the training will be completed only when the necessity for training no longer exists.

In estimating distances, the range to an object which is nearly invisible is very much more difficult correctly to estimate than is the range to a prominent object, and it is generally over-estimated. If the enemy is quite invisible, recourse must be had in estimating the range to some point on the landscape. If the range is given the men by the group leader, the dispersion will not be affected by this factor, but if each man is making and using an individual estimate, then the variety of ranges used will be much greater against a nearly invisible target than against a clearly seen enemy and the resulting dispersion will be correspondingly increased.

Unless the power quickly to recognize an indicated target is well developed, many soldiers of the firing group will fail to pick up the designated objective and will fire either at another object altogether or in the gen-

eral direction only of the designated target. Having an imperfect conception of just what and where the target is, those with poor vision will fire so vaguely and with such varying points of aim that the dispersion will be greatly increased.

It is thus necessary to include in every system of firing instruction an adequate training in the designation and recognition of indefinite targets. It is less trouble to command *"At 1,000 yards, at the enemy,"* when at drill than to designate a definite target, but the training of both troops and officers suffers through the slovenly practice and certainly the preparation of the command for accurate and quick shooting in war has not been advanced.

The size of the dispersion is variable with the training, physical condition and morale of the soldiers. "Training" in marksmanship includes training in aiming, holding, and firing and in addition training in many other things which affect the efficacy of fire, such as visual training, training in estimation of distance, training in fire direction and control, etc., and in the whole subject, that of training in aiming, holding and firing—such training, in other words, as the soldier receives on the target range plays but a minor part.

To those who are content to believe that "parlor shooting" is the one thing necessary to insure effective fire in battle, all theory and study of fire effect is as the idle dream of a pedant. To them, to be able to hit a bullseye on the target range is to be able to hit a man in battle, one merely goes out to the battle field and—"Ready, Aim, Fire," and down goes an enemy. This is repeated as often as necessary, and then the army marches home to dinner. In war things are not so easy

as this, and modern history fails to show that mere skill in shooting exercises any appreciable result. The Boers, than whom no better nation of sharpshooters probably exists today, surprised the world with their shooting—but not because of its excellence since the percent of Englishmen hit in battle was quite small. Their shooting, judged by results in killed and wounded, was about as bad as any on record.*

The idea of the importance of target practice (which unfortunately has blinded many of our best soldiers to the truth), undoubtedly had its origin in the days of the smooth bore musket when the accurate fire of the American woodsman (armed with a *rifle* as against the hostile smooth bore) won for us so many brilliant victories. It should be remembered, however, that the marksmanship of these masters was usually exercised at ranges where the angular errors were paramount and where their skill in aiming consequently was a deciding factor. If armies to-day could march up to within 100 yards of the enemy before deploying and could first open fire at that range or less, it is unquestioned that marksmanship would be the deciding factor, but the world has moved and new factors have entered into the question of fire effect. It is pertinent therefore to investigate these factors to determine whether they should modify our musketry training or not. All of the leading countries of Europe have studied the matter thoroughly and have come to practically one system of musketry in which the mere shoot-

* The Boers by their skillful use of combined fire under the instruction of the German officers, were able to paralyze the English advances in spite of the relatively small physical results of their fire, and thus have acquired a reputation for accuracy of fire which a dispassionate examination of the results of their fire does not sustain.

ing at a target plays a very small part. To continue to build up and augment the importance of target shooting in the face of this change in other countries and that without any investigation into the reasons for their change seems fatuous and stupid.

We have seen how, under the ideal conditions of seeing a well defined target and knowing the exact range, the good marksmen can expect a higher percent of hits than either the average or poor shot, and we shall see too that so soon as these conditions change, especially when the range is unknown, the superior skill of the excellent marksman works to his positive disadvantage. Under the conditions precedent to a discussion of fire effect, however, the effect of skill in marksmanship operates to reduce the size of the dispersion. So soon as the soldier becomes fatigued he begins to increase his angular errors and his dispersion and the same effect is produced by lowering his morale.

THE INFLUENCE OF AN ERROR IN THE DETERMINATION OF THE RANGE.

An error in the determination of the range plays a most important part in deciding the efficacy of fire. In action, the range is usually determined by an estimate or guess; though under certain circumstances an instrumental determination may be possible.

All armies have conducted experiments to determine the probable error in the estimation of the range, and many independent investigations have been made. Probably the most extensive report is that quoted by General von Rohne in his *"Detachment Field Firing of Infantry,"* wherein 2,000 different men made 40,000 estimates at various ranges and established an average error of 20

percent, which corresponds to a probable error of 16.9 percent, or about one-sixth of the range.*

This error is that of "partly trained" men, however, and since proper fire direction presumes that the range is estimated by highly trained experts, it is perhaps too large even for war conditions. A probable error of one-eighth of the range ($12\frac{1}{2}\%$) seems to be a fair assumption for these trained men and is assumed in the various examples worked out in the text. It is certainly conservative and lends itself readily to computations.†

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* The "Proficiency Tests" prescribed in our Small Arms Firing Regulations, provide for the classification of

Sharpshooters	10 percent error.
Marksmen	15 percent error.
First Class	20 percent error.
Second Class	25 percent error.

Conducted as prescribed the "test" has little value and the results, being wholly unrelated to judging distances under service conditions, cannot serve as a guide to probable efficiency even in peace.

† By a *probable error* is meant an error which will be exceeded in one-half of all the cases. Thus, if the target is at 800 yards and the probable error is one-eighth of the range, then 50 out of 100 estimates will place the target between 700 and 900 yards (error 100 yards either way), one-fourth of the estimates will place the target nearer than 700, and one-fourth farther than 900 yards.

		25			50		25		
.4	1.8	6.7	16.1	25	25	16.1	6.7	1.8	.4

400 500 600 700 800 900 1000 1100 1200

True.

This is illustrated in the above figure. Of 100 estimates at a target really 800 yards away there will be

50 which have an error of less than 12.5 percent.

32 which have an error of more than 12.5 percent, but less than 25 percent.

The opportunities for the use of a range finder will be limited in war, and ranges must be guessed to a very great extent. There is no question but that training will lessen the percentage of error in an "estimated range," but even the best of estimators must include a factor of error that is vital to good results in field firing. The absolute amount of this error varies, with the training of the estimators, the character of the ground, target, light, etc., but it conservatively may be placed at one-eighth of the range for trained men and at almost any-

14 which will have an error of more than 25 percent, but less than 37.5 percent.

3 which will have an error of more than 37.5 percent, but less than 50 percent.

1 which will have an error of more than 50 percent.

By using the probable error as a basis of discussion one recognizes that in one-half of all cases this will be exceeded and that the other half will be less than the probable error. If one thinks of the many times when this error will prove too great, he must also think of the equal number of times when it will prove too small. In a very great number of estimates any particular estimate will probably be as great as the "probable error." This probable error should be distinguished from the average error which is always larger (a probable error of 12.5 percent corresponds to an average error of 14.8 percent), for example:

Number of estimate.	Estimate.	Error.	Percent.
1	575	225	28.2
2	680	120	15.0
3	800	0	0.0
4	860	60	7.5
5	986	186	23.3
	Aggregate.	591	74.0
	Average.	118.2	14.8
	Probable.	100	12.5

$$\frac{591}{5} = 118.2 \; ; \; \frac{74.0}{5} = 14.8 \; ; \; 14.8 \times .845 = 12.5 \text{ percent}$$

thing greater than one-eighth of the range for those imperfectly or wholly untrained.

Von Ploennies in his *"New Studies"* states that: "From a personal experience of several years as an instructor of sharpshooters, I have found that, after a special instruction of the men for one year (which was preceded by a twelve months' training in the infantry of the line), of any 100 recorded distances only about sixty are usually estimated with an approximate accuracy of ten percent of error, and only about forty with about five percent; from which it follows that the average accuracy of the estimation is much less, even in time of peace, than 87½ percent, and that the errors in the field will on the average scarcely be confined to 15 percent or 20 percent."

Lieutenant General Parravicino in an article in the *"Revista de Artiglieria"* says that the error of estimation according to his own experiments, amounts to

50 meters at the distance 415 — 530 meters = 10. %.
100 meters at the distance 650 — 750 meters = 14.3%.
150 meters at the distance 850 — 960 meters = 16.5%.

which is an average error of about one-seventh of the true range.

In experiments made by General Rohne with trained men, over unknown ground, the average of errors was found to be

At	520 meters	24.5 percent.
At	1,000 meters	16.4 percent.
At	1,450 meters	6.6 percent.

Of 231 estimates 12 were correct, 69 too great and 150 too small. The average error amounted to one-seventh of the range, but in specific cases it reached as high as 54 percent and in one case 62 percent of the dis-

tance. General Rohne, himself, remarks upon the apparent increase of accuracy at the longer ranges, and accounts for it on the ground that the men passed successively from the shorter ranges by intermediate ranges to the long ones, and were therefore able to use their earlier estimates as a guide at the longer ranges. Not satisfied with the results, he conducted other exercises—this time over known ground—but with precautions against outside influences, and attained an average error of 12.5 percent, that is to say, one-eighth of the range.

The report of the Senior Officer's Course, held at the (English) School of Musketry at Hythe, 1905, gives the results of the tests for all students. Regular officers, estimating at targets, none of which was more distant than 700 yards, made the following errors:

Correct range........	15 percent of estimates.
Within 100 yards....	49 percent of estimates.
Within 200 yards....	20 percent of estimates.
More than 200 yards..	14 percent of estimates.

Commenting upon these results, the report states: "From these figures, compiled at distances under 700 yards, it appears that fire controlled by these officers and non-commissioned officers would be mostly wasted. The standard is about equal to that of 'slightly trained' French officers." * * * French officers and soldiers tested under various conditions made the following mean errors in judging:

Fully trained officers.............	12 percent.
Slightly trained officers...........	20 percent.
Soldiers of the active army........	30 percent.

Remembering that these figures:

Von Ploennies. 1/10
Parravicino. 1/7
Von Rohne. . . . . 1/8.
English 1/5

were attained only by trained estimators, in time of peace, and at silhouettes generally distinct and well lighted, an idea may be formed of the probable error of individual commanders, using their own estimate, rather than a mean of several estimates of trained men, and, estimating over unknown ground, the distance to a moving, dimly seen, suddenly presented target. That the probable error of one-eighth of the range is not excessive must be conceded.*

In order that we may perceive the effect of using an incorrect elevation, it is but necessary to refer to Fig. 44 and its accompanying tables:

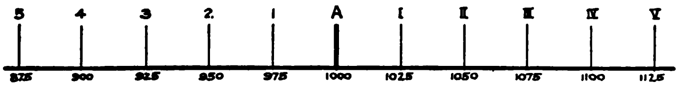

FIG. 44.

If a series of targets is placed 25 yards apart on level ground, as indicated in the figure, each target a paper wall three feet high and the point of aim is taken at the center of the target (line of sight horizontal and elevation 1,000 yards), then average marksmen will expect on the several targets the following number of hits:

*In an experiment made by the author at a true range of 1,300 yards, 32 regular American officers made an average error of exactly 1/8 of the range, the maximum error being 54 per cent.

On Target "A"	26.4 percent of hits.
On Target I	23.2 percent of hits.
On Target II	16.8 percent of hits.
On Target III	10.0 percent of hits.
On Target IV	4.7 percent of hits.
On Target V	2.3 percent of hits.
On Target 1	24.6 percent of hits.
On Target 2	19.6 percent of hits.
On Target 3	12.9 percent of hits.
On Target 4	7.5 percent of hits.
On Target 5	3.1 percent of hits.

"Good Marksmen" will expect—

On Target "A"	40.0 percent of hits.
On Target I	31.0 percent of hits.
On Target II	15.4 percent of hits.
On Target III	4.8 percent of hits.
On Target IV	0.8 percent of hits
On Target V	0.0 percent of hits.
On Target 1	33.5 percent of hits.
On Target 2	19.0 percent of hits.
On Target 3	2.2 percent of hits.
On Target 4	1.5 percent of hits.
On Target 5	0.0 percent of hits.

"Poor Marksmen" will expect—

On Target "A"	13.4 percent of hits.
On Target I	12.6 percent of hits.
On Target II	11.0 percent of hits.
On Target III	9.6 percent of hits.
On Target IV	7.3 percent of hits.
On Target V	6.4 percent of hits.
On Target 1	13.3 percent of hits.
On Target 2	12.5 percent of hits.
On Target 3	11.7 percent of hits.
On Target 4	10.3 percent of hits.
On Target 5	8.9 percent of hits.

Tabulating the foregoing for convenience of comparison,

Class.	875	900	925	950	975	1000	1025	1050	1075	1100	1125
Good. .	.0.0	1.5	2.2	19.0	33.5	40.0	31.0	15.4	4.8	0.8	0.0
Aver. . .	.3.1	7.5	12.9	19.6	24.6	26.4	23.2	16.8	10.0	4.7	2.3
Poor. . .	.8.9	10.3	11.7	12.5	13.3	13.4	12.6	11.0	9.6	7.3	6.4

With this table we can examine the effect of using an incorrect elevation, and the relation between that error and the size of the dispersion. We see that when the target stands at exactly the assumed distance (1,000 yards) the number of hits increases as the dispersion grows smaller. The accurate shooting of the good marksmen gives three times as many hits as the indifferent shooting of the poor marksmen. Indeed, if the men could be so very highly trained as to produce only the dispersion due to the armament, we would expect 89.5 percent of hits on the target assumed. But we also see that the number of expected hits falls off very rapidly with good marksmen and very slowly with poor marksmen (large dispersions) so that with an error of only 50 yards (target at 1,050) the average marksmen would be getting more hits (16.8) than the good marksmen (15.4). As the dispersion of the poor marksmen is still larger, the effect will fall off more slowly, so that against a target 125 yards in error (1,125) where the good marksmen get no hits at all, the poor marksmen will make over 6 percent of hits. If we presume that by training in target practice, the accuracy of the marksmen has been so highly developed that the dispersion of the firers is held down to that inherent in the rifle (15 inches at 1,000 yards), then we would expect:

Class.	875	900	925	950	975	1000	1025	1050	1075	1100	1125
Rifle. . .	..	..	..	1.2	34.8	89.5	28.3	..	..	..	..
Good. . .	.0.0	1.5	2.2	19.0	33.5	40.0	31.0	15.4	4.8	.8	.0
Aver. .	.3.1	7.5	12.9	19.6	24.6	26.4	23.2	16.8	10.0	4.7	2.3
Poor. .	.8.9	10.3	11.7	12.5	13.3	13.4	12.6	11.0	9.6	7.3	6.4

The influence of an inexact range upon these ideally trained marksmen is too apparent to need comment, since an error of only 25 yards in a range of 1,000 (target at 1,025) reduces the number of their hits below that of

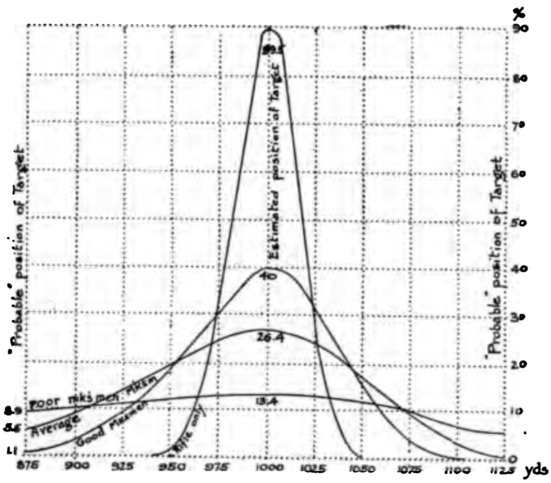

FIG. 45.

Poor marksmen surpass average when error is..........±75 yards
Average marksmen surpass good when error is..........±40 yards
Good marksmen surpass ideal when error is............±25 yards

The probable error at 1,000 yards is ±125 yards, and the "probable" position of the target is therefore at 875 or at 1,125 yards.

average marksmen, while an error of 50 yards utterly destroys their hitting power. This is graphically shown in Fig. 45. Training, therefore, *which seeks only to increase the accuracy of the firers*—to reduce the dispersion—presumes some method of determining the distance with great accuracy; in fact, with an unattainable accuracy in the general case, and hence is fundamentally wrong. The adoption of telescopic sights or other re-

finements which tend to reduce the dispersion are, of course, open to the same objection unless equally efficient instruments are used at the same time to determine the range and the elevation appropriate to the range and meteorological conditions.

It is evident that the effect of an inexact range or elevation will vary in amount with the range, hence a comparison, based on only one range, as that just discussed, may not be generally true.. The best basis upon which to compute the result of an incorrect elevation is that which at the various ranges includes the probable error at that range. The probable error in yards of the estimate of the distance is:

37.5 yards at 300 yards	87.5 yards at 700 yards
50.0 yards at 400 yards	100.0 yards at 800 yards
67.5 yards at 500 yards	112.5 yards at 900 yards
75.0 yards at 600 yards	125.0 yards at 1,000 yards

Using the dispersions of Table II, and computing the density by Table III, the effect of an error of one-eighth of the range is made apparent when we consider the hits obtained from every 100 shots against a wall target 36 inches high at the following distances:

TABLE VI.

With an error of one-eighth of the range, the number of hits obtained from 100 shots against a wall target 36 inches high is:

Range yards.	Error yards.	Number of Hits.		
		Good.	Average.	Poor.
300	37.5	99.0 (99.4)	94.0 (94.7)	62.0 (66.9)
400	50.0	88.8 (93.3)	70.0 (80.9)	47.0 (49.0)
500	62.5	58.8 (79.5)	49.0 (62.3)	32.4 (34.4)
600	75.0	29.4 (66.9)	33.6 (50.0)	23.9 (26.4)
700	87.5	13.3 (58.5)	21.5 (41.1)	18.0 (21.3)
800	100.0	4.5 (51.3)	12.6 (34.4)	13.6 (17.6)
900	112.5	0.9 (45.2)	6.5 (29.9)	10.2 (15.2)
1,000	125.0	0.01 (40.0)	2.4 (26.4)	7.5 (13.4)

NOTE: The numbers in parentheses show the number of hits with correct elevation.

This table shows:

1st. The great advantage of small dispersions (precision in aiming) when the range is exactly known.

2d. That the advantage of small dispersions decreases with the range when the correct elevation is not used, but one differing by only a probable amount from the exact elevation.

3d. That the smaller the dispersion the more rapidly does the effect decrease with an inexact elevation.

4th. That up to 500 yards precision is of more importance than the estimate of the range, because beyond 500 yards the results obtained by good marksmen are less than those attained by average marksmen.

5th. That at 700 yards even poor marksmen will

get more hits than good marksmen. At 800 yards poor marksmen obtain more hits than either of the other classes. At 900 yards poor marksmen will obtain ten times as many hits as good marksmen.

Several deductions may be made from these results. Any comparison of percentages, for example, made at a measured distance, cannot serve as a guide to efficiency in service shooting, because a company of *good* marksmen, whose captain estimates the range incorrectly, may obtain less results than another company of *poor* marksmen whose captain makes the same or even a greater error in his estimation of the range. Again, at ranges less than 500 yards a small percentage is generally due to lack of precision, while at ranges over 700 yards the low percentage is usually the result of an incorrect elevation. A very high degree of training in marksmanship and the use of telescopic or refined sights will still further reduce the hits to be expected in war unless the range is known exactly, or, more correctly speaking, unless the elevation is exactly known, for the smaller the dispersion the greater is the relative influence of the atmospheric factors of heat, barometric pressure, wind, mirage, etc. If one believes that on the battlefield the range can be determined exactly either by guessing or instrumentally, and further that the meteorological conditions can be ascertained and their influence computed, then, under such ideal leadership, the more accurate the fire the greater will be the result. If, on the other hand, one accepts an error of one-eighth as a probable error in the elevation, the deduction is clear that a larger dispersion, *i. e.*, less accurate shooting, is greatly to be desired.

The question naturally arises, "Will our best shots shoot as well in war as in peace?" We have shown that they will not, and that the distinction between the "Ex-

pert Rifleman" and the "Third Class" man will very largely disappear. The size of the war dispersions is taken up and discussed elsewhere; but the foregoing peace dispersions are assumed not in the belief that such small dispersions are probable in war, but to demonstrate the relative importance of two factors in fire efficiency—small dispersions and incorrect elevations. As they are the only dispersions whose size has been determined by actual firing, they alone will serve as a basis for the demonstration.

Another question naturally arises, and that is as to the effect of the range-finder upon the efficiency of infantry fire, and, since all infantry companies, at least in the regular army, will be supplied with range-finders, it is pertinent to inquire into the effect of the use of these telemeters.

Experiments conducted with the greatest care and by trained observers have shown that with our "Pentaprism" or "Weldon" range-finder, ranges *may* be determined with an error not greater than 2 percent so long as the range does not exceed 1,000 yards. Where, however, the base is measured by pacing, this minimum error increases to about 10 percent, and under fire, over rough ground and at an indistinct target, it will be reasonable to assume an error of at least 20 percent, and to expect one even larger. With the new *"Aubry"* range-finders used by our artillery the results are much better, but their use by infantry is hardly practicable. The rifle sight is graduated, except at the long ranges, in spaces representing 50 yards, so that if the range-finder shows a range of 925 yards, the firer must set his sights at 900 or 950 yards, and if an error of 5 per cent has been made by the telemeter (target at 973 yards) the use of the 900-

yard elevation would be an error of 73 yards, or 7.5 percent. But the range does not agree exactly with the elevation which should be used because of the influence upon the projectile of atmospheric conditions. The error when using the range-finder certainly should never equal that to be expected in an estimated range (though in practice it often does), but, because of the quite evident difficulty which will attend its use by an assaulting body of troops, it would seem highly probable that we shall still have to estimate ranges, at least until a practicable range-finder shall have been adopted.* The error of one-eighth assumed in this discussion will apply also to ranges "found" by untrained men and will usually be exceeded in war when estimating is resorted to and often when a range-finder is used.

An error in estimating the range manifests itself in two ways, according as the fire is directed or not. If each of the skirmishers estimates the range for himself and uses the corresponding elevation, then the result will be a wide dispersion of the centers of impact and a corresponding loss in effect of fire. If, on the other hand, all use the same, but an incorrect elevation, the dispersion will remain constant, but the center of impact will not coincide with the center of the object, and a loss of efficiency proportional to the eccentricity of the center of impact will result.

Considering the first case: Assume a target at 800 yards, then one-half of the estimates and centers of impact will lie between 700 and 900 yards; that is to say,

* The *Barr-Stroud* range finder for infantry, recently improved, has been adopted by the French Infantry and is the only infantry range finder which combines reasonable accuracy with practicability. The expense of equipping each company with one of these range finders would not be prohibitive.

an even chance exists that they lie on each side of the true range and that one-half of all the estimates are within the error of one-eighth of the range and one-half exceed that error. The mean dispersion in depth of the centers of impact then is 200 yards, and this corresponds to a vertical dispersion of 114 inches. The mean vertical dispersion at 800 yards when the centers of impact are coincident is 54.30 inches. Applying Didion's law to determine the combined effect of these two sources of error, we get $\sqrt{114^2 + 54.3^2} = 126.5$ inches, as the mean dispersion under the conditions cited.

The effect in hits of this increased dispersion will be to reduce the number of probable hits to less than one-third of what might be expected with average marksmen using the correct elevation (15:51.3). Two factors have entered into the production of this dispersion, one the usual dispersion of average marksmen, which is, generally speaking, a measure of the accuracy of their fire as individuals; the other a dispersion of the centers of impact, which is a measure of the influence of a probable error in the determination of the range. Of these two the latter is more than twice as large as the former, and hence is the controlling factor. We have seen that an error in the elevation is less important at the short ranges; let us now inquire into the effect of the error where the men are individually estimating the range to a target 300 yards distant:

At 300 yards the probable error is 37.5 yards and the angle of fall is 12.2 minutes. The dispersion of the centers of impact therefore is 75 yards $\times$ tan $12.2' =$ 11.4 inches. The mean vertical dispersion of average marksmen at this range is 12.5 inches and the combined effect is $\sqrt{11.4^2 + 12.5^2} = 16.9$ inches. It is apparent that

the controlling factor is the accuracy of the fire (12.5 inches) rather than the incorrect range (11.4 inches). Somewhere, then, between 300 yards and 800 yards we may expect the two factors to be about equal in importance.

At 525 yards the probable error in estimating the distance is 65.5 yards and the angle of fall is 34 minutes. The dispersion of the centers of impact, therefore, is 131.0 yards $\times$ tan $24' = 28.5$ inches. The mean vertical dispersion is 29.6 inches or about the same amount. At ranges under 525 yards the dispersion due to inaccurate aiming is less than that due to inaccurate determination of the range, while beyond 550 yards the dispersion due to an inexact range is the larger of the two. From this point (525 yards) the two influences increase in importance. At the very short ranges the effect of the fire depends almost entirely upon accuracy of aim, while at the very long ranges the accuracy of aiming plays relatively a very insignificant part in determining the effect of the fire. Reference to the following table shows that the better the class of marksmen the shorter the range at which the influence of an inexact elevation becomes dominating in this class of fire:

RELATIVE EFFECT OF AIMING AND DETERMINATION OF RANGE WHEN FIRE IS UNCONTROLLED.

INDIVIDUAL ESTIMATES.

Range Yards	Dispersion of centers	Good	Average	Poor
200	3.0	5.3	7.75	15.5
300	11.4	8.9	12.5	25.0
400	15.3	13.2	18.5	37.0
500	25.0	19.2	27.4	54.8
600	48.0	25.0	36.2	72.4
700	65.0	29.7	45.2	90.4
800	114.0	35.0	54.3	108.6
900	167.0	40.5	63.4	126.8
1,000	201.0	46.5	72.5	145.0

For good marksmen the influences are about equal at 250 yards.

For average marksmen the influences are about equal at 525 yards.

For poor marksmen the influences are about equal at 780 yards.

Considering average marksmen, the two influences are equal, as shown, at about 525 yards; beyond this range, therefore, it is time, energy and money wasted to increase the skill of the individual man unless the development in skill in estimating distances progresses a corresponding amount; and *vice versa,* it is useless to increase the skill in estimating distances unless the skill in marksmanship is correspondingly increased. Knowing the relative power of these two factors, stress must be laid upon reducing that which produces the greatest errors. For example: Assuming that the most effective fire will be delivered at a range of about 500-600 yards, then, from the above table it is evident that as

soon as the marksmen are trained to a "good" grade, the important thing in which to continue their instruction is estimation of distance, whereas with "poor" marksmen, improvement in fire effect must be the result of training in accuracy of fire. The training of the "average" marksman should be along both lines.

So long as the mean dispersion which results from using incorrect elevations is greater than that due to individual marksmanship, as is the case at ranges beyond 525 yards for average marksmen, it is utterly immaterial whether the men shoot well or poorly, and from this it follows that the effect in field firing at the middle and long ranges is influenced to a far greater degree by the estimation of distance than by individual errors in aiming and firing. *Hence, to increase the effect of field firing it is necessary to increase the accuracy of estimating the distances rather than to increase the accuracy of individual fire.*

Before considering the other phase of the question, it is well to observe that such a case as has been assumed above (every man estimating and using his own range) will usually occur at the short ranges. At the longer ranges where the men are still well in hand and the noise of the conflict not too loud, commands designating the elevation will be heard and obeyed, but in action it is certain that the command announcing the elevation will not be heard at ranges under 500 yards, so that at the short ranges where the errors of estimation exercise less effect than do errors in aiming, no bad results will follow the dispersion of the centers of impact *so long as the men are trained to estimate distances with an error not exceeding 15 percent.*

Considering, now, the second case, *i. e.,* the designated elevation used by all, and that elevation incorrect.

If the commander orders an elevation of 1,000 yards, and his estimate of the range, or, better still, his estimate of the elevation, is exactly correct, then the maximum of effect is obtained, the value of this effect being proportional to the skill of the firers (smallness of the dispersion). If, however, an error of one-eighth of the range is assumed (= 125 yards), that is, that the target is really at 875 or at 1,125 yards, then, with a very small dispersion, no result is obtained at all, because the cone of dispersion is so small that none of the shots can reach the target; if, on the other hand, the dispersion is greater, as it assuredly will be, then, with average marksmen, a target wall 36 inches high, at 875 yards or at 1,125 yards, will receive about 2.4 percent of hits. With poorer marksmen producing, say, double the dispersion above considered, six times as many hits will be obtained (14.5). It is apparent that with only the average error of trained estimators, the poorer marksmen will here secure more hits than will the good marksmen. The influence exerted by the estimate of the range in its relation to the character of the marksmen has already been pointed out, and the figures are of the greatest importance since they permit an investigation into the different causes of error and their relative effect. At ranges over 800 yards the maximum of effect is obtained with "experts" *knowing the range exactly,* and the minimum of results is obtained by these same men with the error of one-eighth in estimating the range, which is an error less than will obtain in war with any but the most highly trained estimators, and about what we may expect if we use our present range-finder under service conditions.

The question, then, is fairly presented—"What can and must be done to produce the maximum results under

service conditions?" Target practice alone will not do it, range-finders alone, however accurate and convenient, will not do it, and it is evident that we must either devote more time and attention to range-finding (estimating) so as to bring it up to the high standard of individual excellence which we have established in shooting (on the range only, be it observed), or, contenting ourselves with a mediocre and inferior standard of expected results, spend less time and money on target practice.

A hasty, incorrect but perfectly natural deduction from the foregoing figures is that target practice, *i. e.*, instruction in firing, is useless. A more careful survey of the proposition, however, will show that at ranges less than 525 yards (500 yards in round numbers) the effect of fire depends largely, if not entirely, upon the accuracy of fire. At 400 yards, for example, the "good" marksmen will get more hits, even with an incorrect elevation, than average or poor marksmen using the exact sight. At these shorter ranges, in battle, the firing line will be as dense as is possible with due regard to freedom of motion in handling the rifle, and protection from losses will depend more largely upon a superior fire than upon the cover afforded by the ground. It is here, at the decisive ranges, that skill in shooting will pay for the time and labor of training. A less careful training in marksmanship would deprive the soldier of confidence in his ability to hit the object at which he is firing. On the range he learns not only the elements of aiming, holding and firing, but also is it impressed upon him that he can only hope to make a hit by careful aiming, and the benefit of this is that in battle he will take pains to aim and fire correctly. If, on the other hand, he regards any hit at all as a result of chance, rather than as a result of his

skill, he will scarcely raise the rifle to his shoulder, much less will he make any real attempt to aim.

At the longer ranges the officer must understand that effective fire depends almost wholly upon him, for his estimate of the elevation is more important than the skill in marksmanship of the firers. The rule should be to train men to small dispersions, and officers and non-commissioned officers to small errors in estimating the distance.

It should be evident that individual instruction in shooting can be carried out successfully only at ranges under 500 yards, and the recent suggestion published in a service paper devoted to the interests of shooting, that target practice be conducted at ranges of 1,500 to 2,000 yards, using telescopic sights, is evidently made, not only in ignorance of the conditions which surround war shooting, but is a sad commentary on the development in our country of an intelligent study of fire effect. A great amount of ammunition would be shot away to no purpose and the confidence of the men in their ability to hit would be destroyed. No possible good could come from such practice, and, assuredly, much harm would result.

COMBINED SIGHTS.

In the computation of expected hits at 1,000 yards (page 130, Fig. 45) it was shown that the greater the dispersion the greater the probability of making hits when an inexact elevation is used. We cannot forego the advantages of small dispersions at the short and decisive ranges, and we cannot train our men to small dispersions at the short ranges and large ones at the long ranges, hence, in order assuredly to attain some result, even though it be small, when firing at the longer ranges we